4th Period

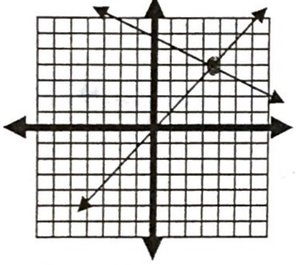
Standards: **A2.A.REI.C.4** (formerly A.REI.C.6) Write and solve a system of linear equations in context.
A2.A.REI.D.6 (formerly A.REI.11) Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology. ★ Include cases where f(x) and/or g(x) are **linear**, polynomial, rational, absolute value, exponential, and logarithmic functions.
Objectives: Students will solve linear systems of equations using tables and graphs, both by hand and calculator

3-1 Solving Systems Using Tables and Graphs

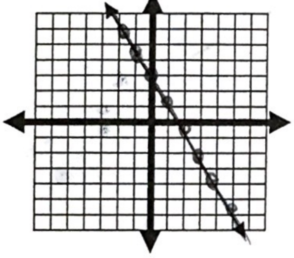
Warm up **Graph/sketch the linear function from standard form (use x and y intercept points)**
$-3x + 4y = 12$

COPY THE Key Concepts

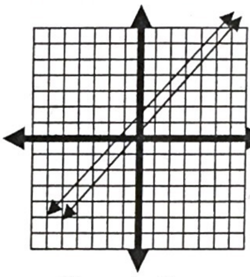
system of equations - a set of two or more equations that use the same variables.
Linear systems - when the graph of each equation of a system is a line.
Solution set – a set of values for the variables that makes all the equations true.
Independent - a system that has a unique (one) solution. (Intersecting lines, different slopes)
dependent - a system that have infinitely many solutions (Coinciding lines, same *m* and same *b*)
inconsistent - a system with no solution (Parallel lines, same *m* but different *b*)



One solution
Consistent
Independent



Infinitely many solutions ✓
Consistent
Dependent



No solution
Inconsistent

$y = mx + b$

1. Classify the system without graphing. $\begin{cases} 4y - 2x = 6 \\ 8y = 4x - 12 \end{cases}$

$4y - 2x = 6$
$4y = 2x + 6$
$\frac{4y}{4} = \frac{2x}{4} + \frac{6}{4}$
$y = \frac{1}{2}x + \frac{3}{2}$

$\frac{8y}{8} = \frac{4x}{8} - \frac{12}{8}$
$\boxed{y = \frac{1}{2}x - \frac{3}{2}}$

inconsistent (NO sol.)

2. Classify the system without graphing. $\begin{cases} y = 3x + 2 \\ -6x + 2y = 4 \end{cases}$ ✓

dependent (inf. many sol.)

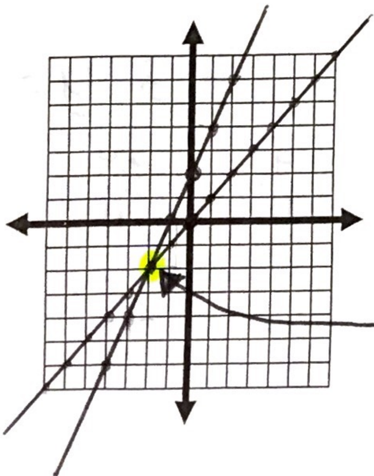
$\frac{2y}{2} = \frac{6x}{2} + \frac{4}{2}$
$y = 3x + 2$

3. Solve each system by graphing and by using a table of values

a. $\begin{cases} y = x \\ y = 2x + 2 \end{cases}$

$y = mx + b$
$y = \frac{1}{1}x + 0$

$y = \frac{2}{1}x + 2$ y-int b

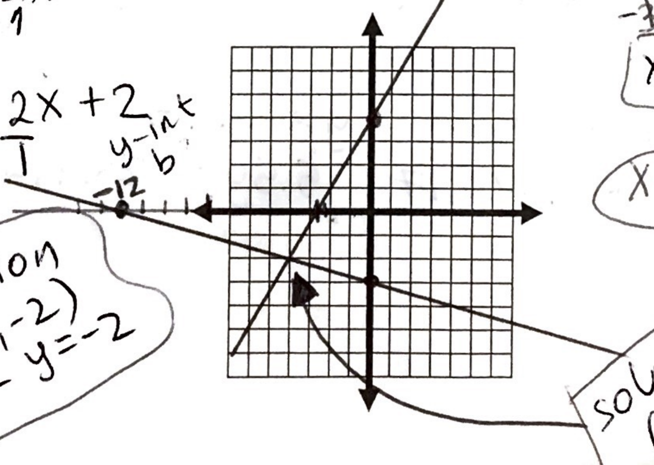


solution $(-2, -2)$ $x = -2$ $y = -2$

b. $\begin{cases} -3x + 2y = 8 \\ x + 4y = -12 \end{cases}$

$\frac{-3x}{-3} = \frac{8}{-3}$ $\frac{2y}{2} = \frac{8}{2}$
$\boxed{x = -2\frac{2}{3} \quad y = 4}$

$x = -12$ $\frac{4y}{4} = \frac{-12}{4}$
$y = -3$

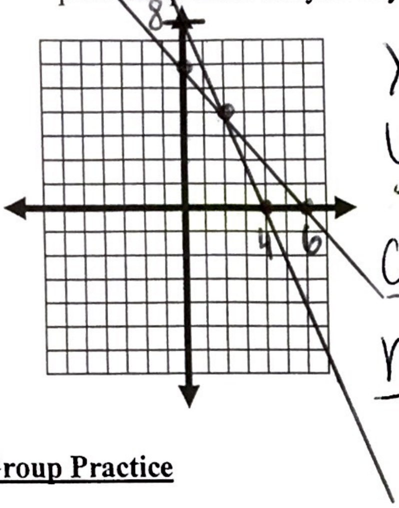


solution $(-4, -2)$ x, y

1

3-1 Continued

4. You bought a total of 6 pens and pencils for \$4. If each pen costs \$1 and each pencil costs \$.50, how many pens and pencils did you buy? Write a system of equations and solve by graphing.



x – number of pens
y – number of pencils
count $x + y = 6$
money $1x + 0.5y = 4$

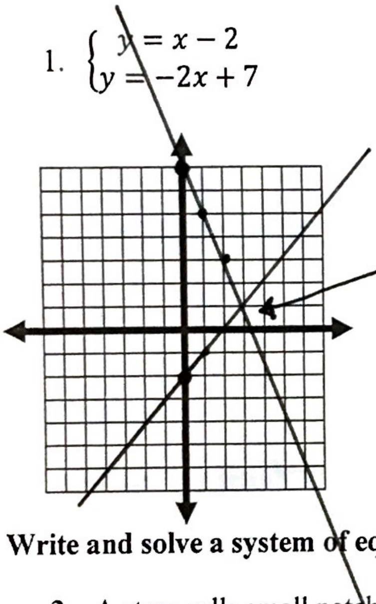
solution
(2, 4)
pens pencils

$\frac{0.5y}{0.5} = \frac{4}{0.5}$
$y = 8$

Group Practice

Solve each system by graphing.

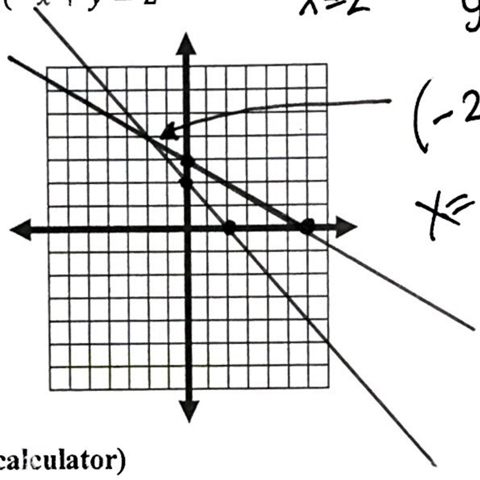
1. $\begin{cases} y = x - 2 \\ y = -2x + 7 \end{cases}$



(3,1)
x=3 y=1

2. $\begin{cases} 2x + 4y = 12 \\ x + y = 2 \end{cases}$

x int: x=6, x=2
y int: y=3, y=2



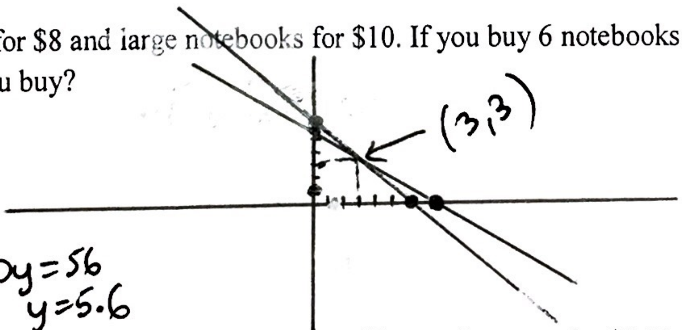
(-2,4)
x=-2 y=4

Write and solve a system of equations by graphing (calculator)

3. A store sells small notebooks for \$8 and large notebooks for \$10. If you buy 6 notebooks and spend \$56, how many of each notebook did you buy?

count $x + y = 6$
money $8x + 10y = 56$

$8x = 56$ $x = 7$
$10y = 56$ $y = 5.6$



4. A shop has one-pound bags of peanuts for \$2 and three-pound bags of peanuts for \$5.50. If you buy 5 bags and spend \$17, how many of each size bag did you buy?

$x = 3$ $y = 2$

count $x + y = 5$ $y = -x + 5$
money $2x + 5.5y = 17$ $5.5y = -2x + 17 \rightarrow y = -\frac{4}{11}x + \frac{34}{11}$

Mini white board activity: PAGE 61 IN THE WORKBOOKS PROBLEMS 1-4, **EXTRA CREDIT #5.**

Exit ticket (use the paper slips in your baskets): Solve a system by graphing y = -3x+ 1 and y= -x -1

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Objectives: Students will solve linear systems of equations using substitution and elimination

3-2 Solving Systems Algebraically

Part 1: Substitution Method

Warm up- Solve the following functions for y

a) -3x -y = 5

-4x +2/3 y = 8

Key Concepts

substitution – means to plug in or replace a variable with an expression.

Steps for Solving Systems using Substitution:

1. Solve one of the functions for x or y, whichever one more convenient
2. Substitute the expression for that function for x or y (whichever one you solved for) in the second function
3. Simplify and solve for the variable
4. Plug in the first answer to one of the original functions to solve for the other variable

Examples

1. Solve the system by substitution. $\begin{cases} y = x \\ y = -x + 2 \end{cases}$

$x = -\cancel{x} + 2$
$+x \quad +\cancel{x}$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{2}{2}$
$\boxed{x = 1}$ $\boxed{y = 1}$

2. Solve the system by substitution. $\begin{cases} x + 3y = 5 \\ -2x + 4y = 0 \end{cases}$

$x = -3y + 5$ $\quad x = -3 \cdot 1 + 5$ $\quad \boxed{x = 2}$

$-2(-3y + 5) + 4y = 0$
$6y - 10 + 4y = 0$
$10y - \cancel{10} = 0$
$+10$
$10y = 10$ $\quad \boxed{y = 1}$

3. Solve the system by substitution. $\begin{cases} r + s = -12 \\ 4r - 6s = 12 \end{cases}$

$r = -s - 12$

$4(-s - 12) - 6s = 12$ $\quad \boxed{s = -6}$ $\quad \boxed{r = -6}$
$-4s - \cancel{48} - 6s = 12$ $\quad r = -(-6) - 12$
$+48$ $\quad 6 - 12 = -6$
$-10s \quad = 60$

3

3.2 Continued

Part 2: Elimination Method

Key Concepts

Elimination - using additive inverses (opposite numbers) and the Addition Property of Equality to cancel a variable.

Steps for Solving Systems using Elimination:

1. Align the corresponding terms above one another (if needed)
2. Multiply one or both equations by a number that will create opposite coefficients with at least one variable (if needed)
3. Add the equations together (cancel the opposite terms and add the resulting terms)
4. Solve for the remaining variable
5. Substitute the value to one of the original values to find the answer to the second variable.

4.Solve the system by elimination. $\begin{cases} 3\cancel{x} + \cancel{y} = -9 \\ -\cancel{3}x - 2y = 12 \end{cases}$

$-y = 3$

$\boxed{y = -3}$

$3x + y = -9$

$3x - 3 = -9$

$+3$

$\frac{3x}{3} = \frac{-6}{3}$

$\boxed{x = -2}$

5. Solve the system by elimination. $\begin{cases} 3x + 5y = 13 \\ 2x + y = 4 \end{cases} \cdot -5$

$3x + 5\cancel{y} = 13$

$-10x - \cancel{5y} = -20$

$\frac{-7x}{-7} = \frac{-7}{-7}$

$\boxed{x = 1}$

$2x + y = 4$

$2 \cdot 1 + y = 4$

$2 + y = 4$

$\boxed{y = 2}$

6. Solve the system by elimination. $\begin{cases} 2x + 4y = -4 \\ 3x + 5y = -3 \end{cases}$ $\cdot 3$, $\cdot -2$

$6\cancel{x} + 12y = -12$

$-\cancel{6}x - 10y = 6$

$\frac{2y}{2} = \frac{-6}{2}$

$\boxed{y = -3}$

$2x + 4y = -4$

$2x + 4(-3) = -4$

$2x - \cancel{12} = -4$

$+12$

$2x = 8$

$\boxed{x = 4}$

7. Solve the system by elimination. $\begin{cases} -6 = 3x - 6y \\ 4x = 4 + 5y \end{cases}$

$(3x - 6y = -6) \cdot 4$

$(4x - 5y = 4) \cdot -3$

$1\cancel{2}x - 24y = -24$

$-12\cancel{x} + 15y = -12$

$\frac{-9y}{-9} = \frac{-36}{-9}$

$\boxed{y = 4}$

$4x = 4 + 5y$

$4x = 4 + 5 \cdot 4$

$\frac{4x}{4} = \frac{24}{4}$

$\boxed{x = 6}$

Systems without unique solutions.

Solve both problems by substitution and elimination.

8. $\begin{cases} -\cancel{3x} + \cancel{y} = -\cancel{5} \\ \cancel{3x} - \cancel{y} = \cancel{5} \end{cases}$ always TRUE $3x - y = 5$

$0 = 0$

infinitely many sol. $\{(2,1), (3,4) \ldots\}$

9. $\begin{cases} \cancel{4x} - \cancel{6y} = 6 \\ -\cancel{4x} + \cancel{6y} = 10 \end{cases}$

$0 = 16$ NEVER TRUE

NO solutions!

Mini white board activity: PAGE 65 IN THE WORKBOOKS PROBLEMS 1-4, **EXTRA CREDIT #5.**

Exit ticket (use the paper slips in your baskets): Solve the system using a method of your choice

-x +y = 10 and 3y= 6x+18

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Objectives: Students will solve linear systems of inequalities by graphing.

3-3 Solving Systems of Inequalities

Key Concepts

system of inequalities - a set of two or more inequalities that use the same variables.

Steps to Solving Systems of Inequalities by Graphing:

1. Solve both inequalities for y 2. Graph both inequalities (use dashed line for < or >, or full line for ≤ or ≥)

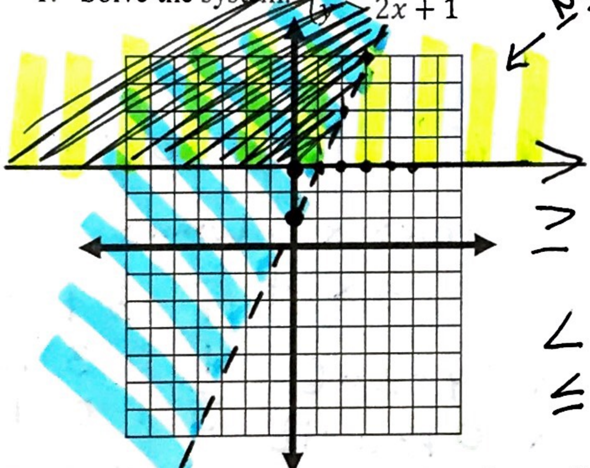
3. Determine at which region two inequalities overlap and shade that area of the graph

Examples

1. Solve the system. $\begin{cases} y \geq 3 \\ y > 2x + 1 \end{cases}$

≥ ≤

NOT included >, <

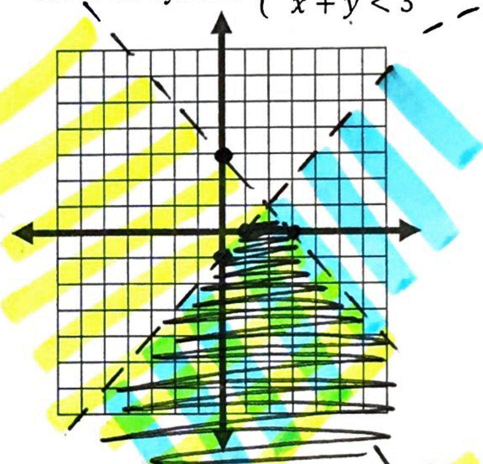


≥ shade above

< ≤ shade below

Solve the system. $\begin{cases} -x + y < -1 \\ x + y < 3 \end{cases}$

below



THINK question: Can a system of two inequalities have no solutions? Explain.

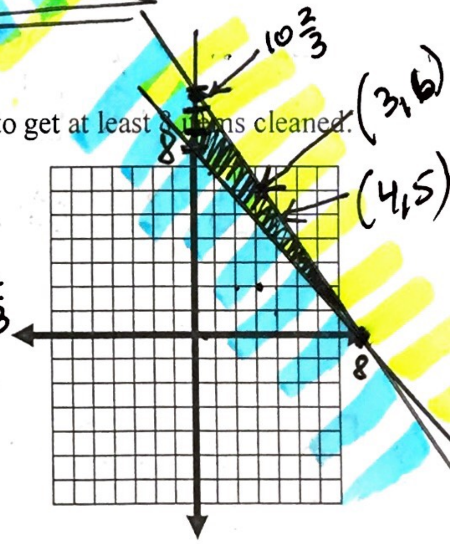
3. The dry cleaner charges $4 to clean a pair of pants and $3 to clean a shirt. You want to get at least 8 items cleaned. You have $32 to spend on dry cleaning.

Write a system of inequalities to model the situation and solve the system by graphing.

x – pants at least

y – shirts $x + y \geq 8$ count $\frac{32}{3} = 10\frac{2}{3}$

$4x + 3y \leq 32$ money



Mini white board activity: PAGE 69 IN THE WORKBOOKS PROBLEMS 1-4, **EXTRA CREDIT #5.**

Exit ticket (use the paper slips in your baskets): solve the system of inequalities by graphing $\begin{cases} \quad + y < 5 \\ \quad < 3x - 2 \end{cases}$

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Objectives: Students will solve linear systems of equations using matrices.

3-6 Solving Systems Using Matrices

Warm up **Solve the system of equations using any method learned in previous lessons (graphing, ~~substitution~~, elimination)**

$-x + y = 5$
$7 = 2y - 5x$

$y = x + 5$

CALculator

$\frac{5x}{2} + \frac{7}{2} = \frac{2y}{2}$

$\boxed{x=1 \quad y=6}$

$(-x + y = 5) \cdot 5$
$-5x + 2y = 7$

$5x - 5y = -25$
$-5x + 2y = 7$

$-3y = -18$

$\boxed{y=6}$

$-x + 6 = 5$
$-x = -1$

$\boxed{x=1}$

Key Concepts

MATRIX - a rectangular array of numbers ; Element - each number in a matrix

Examples

1. Consider the matrix $A = \begin{bmatrix} 4 & 9 & 17 & 1 \\ 0 & 5 & 8 & 6 \\ 3 & 2 & 10 & 0 \end{bmatrix}$. How many rows does matrix A have? 3 ROWS. How many columns does matrix A have? 4 columns. How many elements does matrix A have? 12. What is the dimension of matrix A? ROWS x columns 3 x 4. Identify elements a_{23} and a_{14}. 8 1

2) Write the system as a matrix then solve it.

a. $\begin{cases} -4x - 2y = 7 \\ 3x + y = -5 \end{cases}$ $x = -3/2$ $y = -1/2$

$\begin{bmatrix} -4 & -2 & 7 \\ 3 & 1 & -5 \end{bmatrix}$ solution $\begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & -1/2 \end{bmatrix}$

b. $\begin{cases} x - 3y + z = 6 \\ x + 3z = 12 \\ y = -5x + 1 \end{cases}$

$x - 3y + z = 6$
$x \; 0 + 3z = 12$
$5x + y \; 0 = 1$

$\begin{bmatrix} 1 & -3 & 1 & 6 \\ 1 & 0 & 3 & 12 \\ 5 & 1 & 0 & 1 \end{bmatrix}$

$x = 15/47$
$y = -28/47$
$z = \frac{183}{47}$

One solution answer example: $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $x=3$ $y=2$

No solution answer example: $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ $0 = 2$ Never TRUE sol.

Infinitely many solutions answer example: $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $x + y = 3$ $0 = 0$ ✓ always TRUE

Group work Solve the system using a matrix.

a) $\begin{cases} 3x + 4y = 12 \\ 2x + y = 10 \end{cases}$

$\begin{bmatrix} 3 & 4 & 12 \\ 2 & 1 & 10 \end{bmatrix}$ sol $\begin{bmatrix} 1 & 0 & \frac{28}{5} \\ 0 & 1 & -\frac{6}{5} \end{bmatrix}$

$x = 28/5$ $y = -\frac{6}{5}$

b. $\begin{cases} 2x - y + z = 4 \\ x + 3y - z = 11 \\ 4x + y - z = 14 \end{cases}$ $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & -1 & 1 & 4 \\ 1 & 3 & -1 & 11 \\ 4 & 1 & -1 & 14 \end{bmatrix}$ $x=3$ $y=3$ $z=1$

c) $\begin{cases} 2x + 3y - 2z = -1 \\ x + 5y = 9 \\ 4z - 5x = 4 \end{cases}$

$2x + 3y - 2z = -1$
$x + 5y = 9$
$-5x + 4z = 4$

$\begin{bmatrix} 2 & 3 & -2 & -1 \\ 1 & 5 & 0 & 9 \\ -5 & 0 & 4 & 4 \end{bmatrix}$ Sol. $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$ $x=4$ $y=1$ $z=6$

Mini white board activity: PAGE 81 IN THE WORKBOOKS PROBLEMS 1-3, **EXTRA CREDIT #4.**

Exit ticket (use the paper slips in your baskets): Solve the system using a matrix $\begin{cases} 3x - y = -1 \\ 4x + y = 3 \end{cases}$