Standard(s): **A2.N.RN.A.1** (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Objective(s); **Students will be able to simplify expressions using properties of rational exponents**

Pre-requisite skills for chapter 6: Review of Properties of Exponents

Warm Up

Write each number as a square of a number.

1. 25 5^2 $(-5)^2$

2. 0.09 0.3^2 $(-0.3)^2$

Write each expression as a square of an expression.

3. x^{10} $(\quad)^2 = x^{10}$

4. $169x^6y^{12}$

Key Concepts and examples for each key concept

Property	Rule with exponent	Rule with words
Power of One a^1	a	Always base
Negative Power a^{-m}	$\frac{1}{a^m}$ reciprocal	Flip sign of exponent, Flip base
Power of Zero a^0	1	Always 1
Power of a Product $a^m \times a^n$	a^{m+n}	Keep base add exponents
Power of a Power $(a^m)^n$	$a^{m\cdot n}$	Keep base multiply exponents
Power of a Quotient $\frac{a^m}{a^n}$	a^{m-n}	Keep base subtract exponents

1. $5^1 = 5$ $x^1 = x$

2. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $x^{-3} = \frac{1}{x^3}$

3. $100{,}000^0 = 1$ $(3x^2)^0 = 1$

4. $x^5x^3 = x^8$ $x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x$ $x^{-10}x^7 = x^{-3} = \frac{1}{x^3}$

5. $(2x^5)^3 = 2^3x^{15}$ not exponent coefficient $= 8x^{15}$ $(x^{-10})^7 = x^{-70} = \frac{1}{x^{70}}$

6. $\frac{x^6}{x^2} = x^{6-2} = x^4$ $\frac{x^6}{x^{-3}} = x^{6-(-3)} = x^9$

7. $(xy^2)^3 = x^3y^6$ $(x^{-1}y^3)^4 = x^{-4}y^{12} = \frac{1}{x^4}y^{12} = \frac{y^{12}}{x^4}$

One more property: $a^mb^m = (ab)^m$

Examples

1. Simplify and rewrite each expression using only **positive integers**.

a. $\frac{(x^2y)^0}{2x^{-3}}$ b. $(3a^4)(-2a^{-5})$ c. $\frac{6a^3b^{-2}c^5}{ab^{-3}c^2}$

d. $(-3x^{-3}y^4)^2$ e. $\left(\frac{2x^3y^{-2}}{3}\right)^3$ f. $\left(\frac{3r^{-2}s^3t^0}{3rs}\right)^{-3}$

2. On a separate sheet of paper, CREATE 6 problems (one for each property) involving coefficients and powers, then pass it on to your group member to your right to simplify it. Once you solve it, pass it on to the next person to your right to check the work. You may use the examples from the key concepts to guide you.

Exit ticket: Simplify using exponent properties and write the answers using positive integers only $(3x^{-2})/(2x^{-1})$

Period 1

Standard(s): A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Objective: Students will be able to find n^{th} roots of expressions

Section 6.1: Roots and Radical Expressions

Warm Up
Simplify.

4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

1. $\sqrt{48} = \sqrt{16 \cdot 3}$ $4\sqrt{3}$

2. $\sqrt{12}\sqrt{3} = \sqrt{12 \cdot 3}$ $\sqrt{36} = \boxed{6}$

3. $\sqrt{\frac{16}{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{4\sqrt{5}}{5}}$

RATIONALIZING the denominator

Key Concepts

nth ROOT - For any real numbers *a* and *b*, and any positive integer *n*, if $a^n = b$, then *a* is an *n*th root of *b*.
If *n* is odd, there is one real *n*th root. If *n* is even, there are two real *n*th roots.
+ or − +and −

index radical sign $\sqrt[n]{a}$ radicand

RADICAND - the number under the radical.
index - the degree of the root.
PRINCIPAL ROOT - the positive root when the number has two real roots.

$3^4 = 81$ 3 is the 4th root of 81

Examples

1. Find all the real cube roots of:

a. 0.027 $\sqrt[3]{0.027} = 0.3$

b. -125 $\sqrt[3]{-125} = -5$

c. 1/64 $\sqrt[3]{\frac{1}{64}} = \frac{1}{4}$

Find all the fourth roots of: even +, −

a. 625 $\sqrt[4]{625} = \pm 5$

b. -0.0016 $\sqrt[4]{-0.0016}$ = not real $\pm\left(\frac{\sqrt{2}}{10} + \frac{\sqrt{2}}{10}i\right)$

c. 81/625 $\sqrt[4]{\frac{81}{625}} = \pm\frac{3}{5}$

2. What is each principal real-number root?

a. $\sqrt[3]{-27}$ -3

b. $\sqrt[2]{0.09}$ 0.3

c. $\sqrt[4]{-16}$ not REAL even root of a negative

d. $\sqrt{(-3)^2} = \sqrt{9}$ $|-3| = 3$

Key Concepts

*n*th Roots of *n*th Powers
For any real number *a*, $\sqrt[n]{a^n} = \begin{cases} a \text{ if n is odd} \\ \lvert a\rvert \text{ if n is even} \end{cases}$

$\sqrt[3]{(-2)^3} = -2$

even $\sqrt{(\ )^{even}}$ Need | |

$\sqrt[4]{(-2)^4} = |-2| = \boxed{2}$

Examples

1. Simplify each radical expression.

a. $\sqrt[3]{27a^3b^3}$

3ab

b. $\sqrt[4]{x^{16}y^4}$ even even even root!

$x^4|y|$

classwork pg 157

Period 5

Standard(s): A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Objective: Students will be able to perform operations (add, subtract, multiply, divide) radical expressions

Section 6-2 and 6-3: Operations with Radical Expressions

Warm Up $2^3=8$ $3^3=27$ $4^3=64$ $5^3=125$ $6^3=216$ $7^3=343$

Simplify each and find the principal real roots:

1) $\sqrt{150}$ $=\sqrt{25\cdot 6}$ $=5\sqrt{6}$

2. $\sqrt{288}$ $\sqrt{144\cdot 2}$ $12\sqrt{2}$

3. $\sqrt[3]{-81}$ $=\sqrt[3]{-27\cdot 3}$ $=-3\sqrt[3]{3}$

4. $\sqrt[4]{-16}$ even – NOT REAL

5. $\sqrt{16x^2y^4z^6}$ $4|x|y^2z^3$

Key Concepts

Combining Radical Expressions: Product	Combining Radical Expressions: Quotient
If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$.	If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$.

RATIONALIZE denominator - rewriting an expression so that there are no radicals in the denominator and no fractions in any radical expression.

Examples

1. Can you simplify the product of the rational expressions? If yes, simplify, if not state why not.

a. $\sqrt{18}\cdot\sqrt{2}=\sqrt{18\cdot 2}$ $=\sqrt{36}=6$

b. $\sqrt{7}\cdot\sqrt[3]{2}$ cannot multiply

c. $\sqrt[3]{4}\cdot\sqrt[3]{-2}$ $=\sqrt[3]{-8}=-2$

2. Simplify $\sqrt[3]{54x^7}$

$\sqrt[3]{27\cdot 2\, x^6\cdot x}$

$3x^2\sqrt[3]{2x}$

3. Multiply and Simplify $\sqrt[3]{25xy^8}\cdot\sqrt[3]{5x^4y^3}=\sqrt[3]{125x^5y^{11}}$

$=5\sqrt[3]{x^3x^2y^9y^2}=5xy^3\sqrt[3]{x^2y^2}$

4. Divide and simplify.

a. $\frac{\sqrt[3]{-81}}{\sqrt[3]{3}}$ $\sqrt[3]{-27}$ $=-3$

b. $\frac{\sqrt[3]{192x^8}}{\sqrt[3]{3x}}$ $\sqrt[3]{64x^7}=4x^2\sqrt[3]{x}$ x^6 x

c. $\frac{\sqrt{x^3}}{\sqrt{3x^2y}}$ $\sqrt{\frac{x}{3y}}\cdot\frac{\sqrt{3y}}{\sqrt{3y}}=\frac{\sqrt{3xy}}{3y}$ RATIONALIZING denom.

d. $\sqrt[3]{\frac{5}{4y}}\cdot\frac{\sqrt[3]{2y^2}}{\sqrt[3]{2y^2}}=\frac{\sqrt[3]{10y^2}}{2y}$

LIKE RADICALS - radical expressions that have the same index and the same radicand.

Rationalize denominator - multiply the numerator and denominator of the fraction by the conjugate of the denominator. $a+\sqrt{b}\leftrightarrow a-\sqrt{b}$

Examples

1) a. $3\sqrt{xy}+7\sqrt{xy}$ $10\sqrt{xy}$

b. $3\sqrt[3]{x}-2\sqrt[3]{3x}$ cannot subtract

c. Simplify $3\sqrt{20}-\sqrt{45}+4\sqrt{80}$ (4 5, 9 5, 16 5)

$3\cdot 2\sqrt{5}-3\sqrt{5}+4\cdot 4\sqrt{5}$

$6\sqrt{5}-3\sqrt{5}+16\sqrt{5}=19\sqrt{5}$

2) a. Multiply $(2+4\sqrt{3})(1-5\sqrt{3})$

$2-10\sqrt{3}+4\sqrt{3}-20\sqrt{3^2}=-58-6\sqrt{3}$

Rationalize the denominator $\frac{2-\sqrt{3}}{4+\sqrt{3}}\cdot\frac{4-\sqrt{3}}{4-\sqrt{3}}$ $=\frac{8-2\sqrt{3}-4\sqrt{3}+3}{4^2-\sqrt{3^2}}$ $\frac{11-6\sqrt{3}}{13}$

Classwork: Workbook pages 161 and 165. Show all your work on a separate sheet of paper!!!

Exit ticket:

Standard(s): A2.N.RN.A.1 (formerly N-RN.A.1) Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Objective: Students will be able to convert expressions from radical to rational exponents form and use it to simplify expressions and solve real life problems.

Section 6.4 : Rational Exponents

$b^3 \cdot b^4 = b^7$

Warm Up

Simplify.

1. $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$
2. $(3x)^{-2} = \frac{1}{(3x)^2} = \frac{1}{9x^2}$
3. $(5x^2y)^{-3} = \frac{1}{(5x^2y)^3} = \frac{1}{125x^6y^3}$
4. $(2a^{-2}b^3)^4$ $16a^{-8}b^{12}$

Key Concepts

Rational Exponent fraction in the exponent

If the nth root of a is a real number, m is an integer and m/n is in lowest terms, then

$a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ power ROOT

$a^{\frac{1}{2}} = \sqrt{a}$ $a^{\frac{1}{4}} = \sqrt[4]{a}$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $a^{\frac{1}{10}} = \sqrt[10]{a}$ $a^{\frac{4}{3}} = \sqrt[3]{a^4}$ $a^{\frac{2}{3}} = \sqrt[3]{a^2}$

Examples

1) Simplify.

a. $64^{\frac{1}{3}}$ $= \sqrt[3]{64} = 4$

b. $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}} = 7^1$ $\sqrt{7} \cdot \sqrt{7} = 7$

c. $5^{\frac{1}{4}} \cdot 125^{\frac{1}{4}}$ $(5 \cdot 125)^{\frac{1}{4}}$ $(5^1 \cdot 5^3)^{\frac{1}{4}}$ $(5^4)^{\frac{1}{4}} = 5^1 = 5$

2) Convert to radical form.

a. $x^{\frac{3}{7}}$ $\sqrt[7]{x^3}$

b. $y^{-3.5}$ write as a fraction $y^{-\frac{7}{2}} = \frac{1}{y^{\frac{7}{2}}} = \frac{1}{\sqrt{y^7}}$ do not need to write 2

3) Convert to exponential form.

a. $\sqrt{a^5}$ $a^{\frac{5}{2}}$

b. $\sqrt[5]{b^3}$ $b^{\frac{3}{5}}$ ← power ↑ root

c. $\sqrt[3]{3x^2}$ 2 applies to x only! $(3x^2)^{\frac{1}{3}}$ $3^{\frac{1}{3}}x^{\frac{2}{3}}$

d. $\sqrt[3]{(3x)^2}$ applies to both 3 and x! $(3x)^{\frac{2}{3}}$ $3^{\frac{2}{3}}x^{\frac{2}{3}}$

Key Concepts

All the properties of **integer** exponents also apply to **rational** exponents.
(See properties of exponents from Section 6.0)

Examples

Rewrite in exponential form!

1) Write $\frac{\sqrt[4]{x^3}}{\sqrt[8]{x^2}}$ in simplest form. $= \frac{x^{3/4}}{x^{2/8}} = \frac{x^{3/4}}{x^{1/4}} = x^{3/4-1/4} = x^{2/4} = x^{1/2} = \boxed{\sqrt{x}}$

2) Simplify each number.

a. $(-27)^{\frac{2}{3}}$ ← power (second) ← root (first) $(-3)^2 = 9$

b. $25^{-2.5}$ $25^{-5/2} = \frac{1}{25^{5/2}} = \frac{1}{5^5} = \boxed{1/3125}$

c. $(243a^{-10})^{\frac{2}{5}}$ in simplest form. $3^2a^{-4} = \boxed{\frac{9}{a^4}}$

~~Classwork: Workbook pages~~ Show all your work at the back of this paper!!!

Exit ticket: 169 (take pictures of or copy)

1st

2.A.REI.A.2 (formerly A-REI. A.2) Solve rational and radical equations in one variable and identify extraneous solutions when they exist. **A2.A.REI.D.6** (formerly A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology. **A2.A.REI.A.1** (formerly A-REI. A.1) Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A2.A.CED.A.1** (formerly A-CED.A.1) Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and **rational** and exponential functions. **A2.A.CED.A.2** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
Objectives: Students will apply the knowledge of rational exponents to solve real life problems involving radical equations.

6-5 Solving Square Root and Other Radical Equations

Warm up – use your prior knowledge to answer these questions

1. Simplify
a) $27^{2/3}$
cube root of 27 = 3
$3^2 = 9$

b) $36^{1/2}$
square root
6, or -6

2. Solve the equation and check for extraneous solutions.
$-(x+2)^2 + 5 = -4$
$-5 \quad -5$
$\sqrt{(x+2)^2} = \pm\sqrt{9}$
$x + 2 = 3$, $-2 \quad -2$, $\boxed{x = 1}$
$x + 2 = -3$, $-2 \quad -2$, $\boxed{x = -5}$

Check: plug in
$-(1+2)^2 + 5 = -4$
$-9 + 5 = -4$ ✓
$-(-5+2)^2 + 5 = -4$
$-9 + 5 = -4$ ✓

I check when x appears in multiple places in an equation

Key Concepts

RADICAL EQUATION – an equation that has a variable in a radicand or has a variable with a rational exponent.
$\sqrt[3]{x+1}$ $\qquad (x+1)^{2/3}$

It is possible to get extraneous solutions for square root and other radical equations, so check when x appears at multiple places in an equation.

Examples

1. $-7 + \sqrt{2x+1} = 3$
$+7 \qquad +7$
$(\sqrt{2x+1})^2 = 10^2$
$2x + 1 = 100$
$-1 \quad -1$
$\frac{2x}{2} = \frac{99}{2}$
$\boxed{x = 49.5}$

2. $4(x-2)^{\frac{3}{5}} = 32$
$\div 4 \qquad \div 4$
$\left((x-2)^{\frac{3}{5}}\right)^{\frac{5}{3}} = 8^{\frac{5}{3}}$
power, ROOT, RECIPROCAL of $\frac{3}{5}$
$x - 2 = 2^5$
$\underbrace{2 \cdot 2}_{4} \cdot \underbrace{2 \cdot 2}_{4} \cdot 2$
$x - 2 = 32$
$+2 \quad +2$
$x = 34$

Solve. Check for extraneous solutions. Use your calculators if necessary to find the intersect, or roots of your equations.

3. $\sqrt{x-3} - \cancel{1} = 2x$

$+1 \quad +1$

$2x \; +1$

$2x$

$+1$

$\cancel{\sqrt{}}\, x-3 = (2x+1)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$\cancel{x} - \cancel{3} = 4x^2 + 4x + 1$

$-\cancel{x} + \cancel{3} \qquad -x + 3$

$0 = 4x^2 + 3x + 4$

Calculator: no solutions (no x-int)

Challenge: $\sqrt{2x+1} - \sqrt{x} = 1$

5. Real life problem- For the meteor crater in Arizona, the formula $d = 2\sqrt[3]{\frac{V}{0.3}}$ relates the diameter d of the rim (in meters) to the volume V (in cubic meters). What is the volume of the Meteor Crater if the diameter is 1.2 km?

Group work: Mini – white board activity for lesson 6-5 Standardized test prep page 173, question 1-4

Show all work!!!

1) 2) 3) 4)

Summarize the key concepts you learned today. Use the standards and objectives to help you with vocabulary. These concepts will be covered on your test on November 30th to check for your mastery of the standards.

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Extra credit: The challenge question #3, problem #5 on page 173, and Think About the Plan page 170

Exit ticket: Solve $(6x-5)^{\frac{1}{3}} + 3 = -2$