Standards: **A2. F.LE.A.1** (formerly F-LE.A.2) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or input-output pairs. **A2. F.LE.B.3** (formerly F-LE.B.5) Interpret the parameters in a linear or exponential function in terms of a context. **A2.F.IF.B.3** Graph functions expressed symbolically and show key features of the graph, by hand and using technology. c. Graph exponential and logarithmic functions, showing intercepts and end behavior. **A2.F.IF.B.5** (formerly F-IF.C.9) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **A2. F.IF.A.2** (formerly F-IF.B.6) Calculate and interpret the average rate of change of a function (presented symbolically) Estimate the rate of change from a graph. **A2.A.REI.D.6** (formerly A-REI.D.11) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the approximate solutions using technology. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **A2.F.BF.B.3** (Formerly F-BF.B.3) Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Objectives: Students will be able to construct and exponential functions, by hand and casio calculators. Students will calculate and interpret rate of change of a function, and apply these concepts to real life problems.

Section 7.1: Exploring Exponential Models

Warm Up Evaluate each expression for the given value of x.

1. 2^x for $x = 3$
2. 2^{3x+4} for $x = -1$
3. $(1/2)^x$ for $x = 0$
4. Solve $3^x = 81$

5. Number 5 is raised to some power. Can the result ever equal zero? Why?

Prerequisite knowledge: RATE OF CHANGE (OR SLOPE) = CHANGE IN Y / CHANGE IN X

Key Concepts

Exponential function - a function with the general form $y=ab^x$ where x is a real number, $a \neq 0$, $b > 0$, and $b \neq 0$.

TIME

initial value y-int

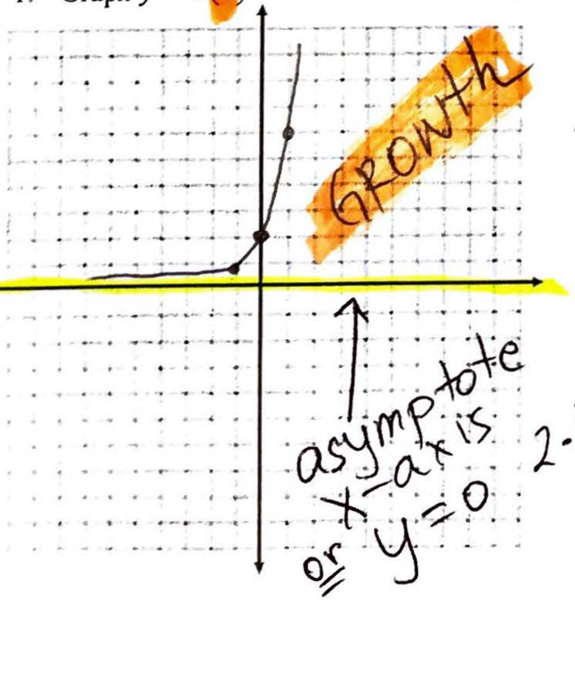
decay or growth factor

Exp. growth - when $b > 1$ Exp. decay - when $0 < b < 1$

asymptote - a line that a graph approaches as x or y increases in absolute value. EX $\frac{2}{3}$

Examples

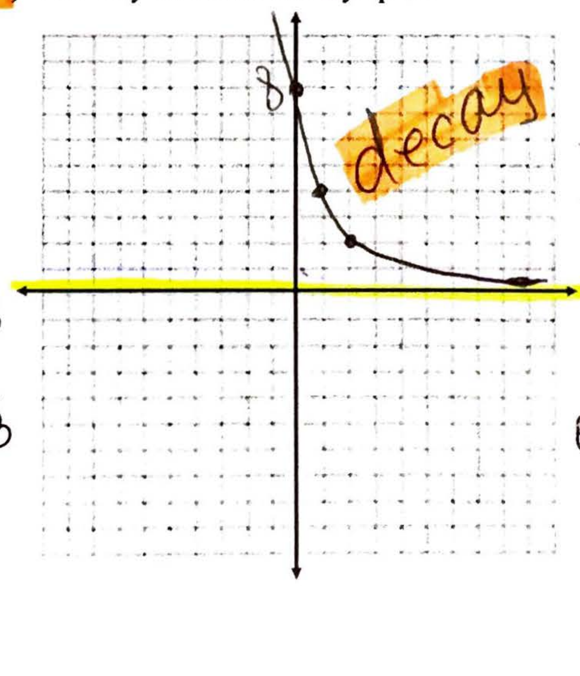
1. Graph $y = 2(3)^x$ y-int



X	y
0	2
1	6
2	18
-1	2/3

$2 \cdot 3^{-1} = 2 \cdot \frac{1}{3}$

2. Graph $y = 8(0.5)^x$. Identify the horizontal asymptote.



X	y
0	8
1	4
2	2
-1	16

$0.5^2 = 0.25$
$= \frac{1}{4}$

2. Without graphing, determine whether the functions represent exponential growth or decay.

   a. $y = 3(\frac{2}{3})^x$ $\frac{2}{3} < 1$ decay   b. $y = 0.25(2)^x$ GROWTH $2 > 1$

3. You invest \$1000 into a college savings account for 4 years, the account pays 5% interest annually. What is the amount in your bank account at the end of the fourth year? If you decide to save for 10 more years, what is the amount you saved?

$y = a\,b^{x \leftarrow 4}$ $a = 1000$, $b = 1.05$

$100\% + 5\% = 105\% = \boxed{1.05}$ GROWTH factor

$y = 1000 \cdot 1.05^4 = \boxed{1215.61}$

$y = 1000 \cdot 1.05^{14} = \boxed{1979.93} \rightarrow$ SAVED 979.9

4. The population of the Iberian lynx is 150 in 2003 and is 120 in 2004. If this trend continues and the population is decreasing exponentially, how many Iberian lynx will there be in 2014?

Interesting fact: "The Iberian lynx is a wild cat species native to the Iberian Peninsula in southwestern Europe that is listed as Endangered on the IUCN Red List. It preys almost exclusively on the European rabbit"

$A = P_i e^{rt}$

2003	150
2004	120
2014	

$\frac{120}{150} = \frac{150}{150}$

$0.8 =$

5. Your parents deposited \$1500 in a savings account at the end of 4th grade. The account pays 4.5% annual interest. How much money will be in the account at the end of 12th grade (volunteer 1)? How much money have you saved by opening a savings account (what is the interest) (volunteer 2)? How many more years has to pass after 12th grade for the amount in the account to be \$3617.57 (set up an equation only- volunteer 3)?

Mini-white board activity- Page 189

Please write the question down and the explanation to how you knew what the answer is.

1)   *2)*   *3)*   *4)*

Summarize the key concepts you learned today. Use the **standards and objectives** to help you with vocabulary words. These concepts will be covered on your test on December 14th to check for your mastery of the standards (6-6 to 6-8 and 7-1 to 7-4).

____

____

____

____

Challenge/Extra credit: Think about a plan page 186 or TEXTBOOK PAGE 421 TASK 1

Exit ticket: Graph the function $y = 2(4)^x$. Use the GRAPH PAPER that's in your baskets!

Standards: *see lesson 7-1*

Objectives: Students will be able to graph transformed functions in the form $y = ab^x$, where b can be any positive real number or number **e**, and apply them to real life problems involving continuous functions.

Section 7.2: Properties of Exponential Functions

Warm Up

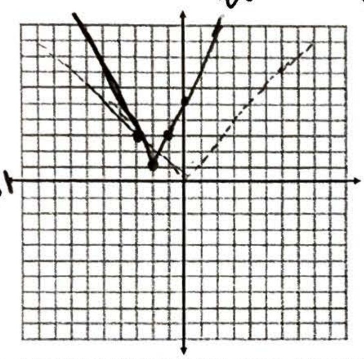
Write an equation for each translation, then graph the transformed function (+) (-)

1. $y = |x|$, 1 unit up, 2 units left, stretch by factor 2

$a = \frac{1}{1} \rightarrow \frac{2}{1}$

$y = 2|x+2|+1$

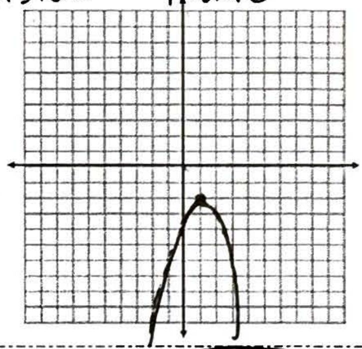
L 2 up 1



2. $y = x^2$, 2 units down, 1 unit right, reflection over the x axis.

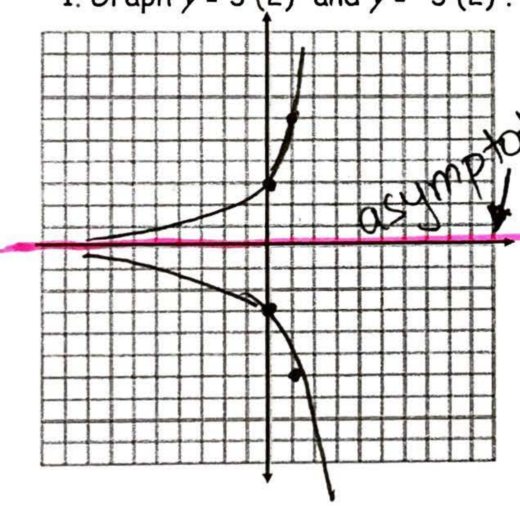
outside inside Negative

$y = -(x-1)^2 - 2$



Examples

1. Graph $y = 3(2)^x$ and $y = -3(2)^x$.



asymptote

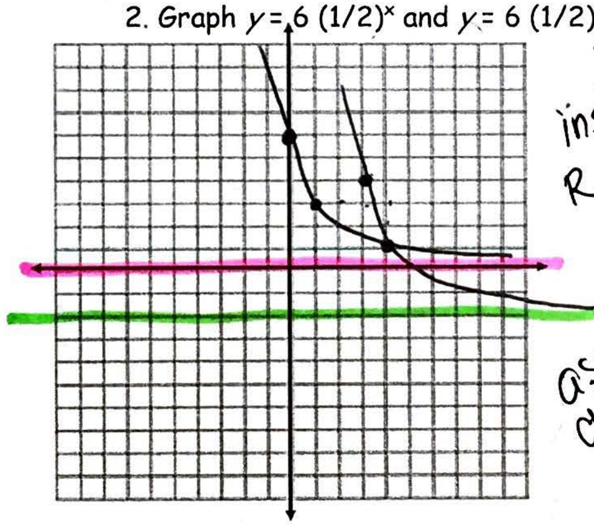
reflection!

$y \neq 0$

2. Graph $y = 6(1/2)^x$ and $y = 6(1/2)^{x-3} - 2$.

inside R 3

outside down 2

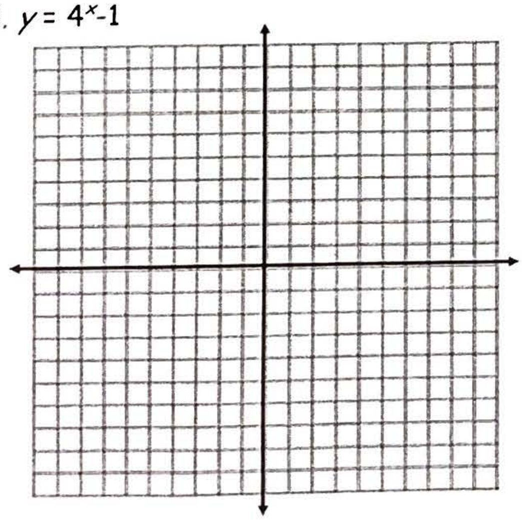


asymptotes also shift!

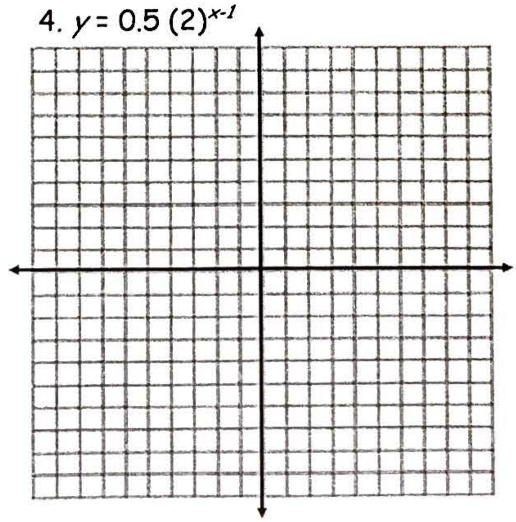
Group practice

Graph each function.

3. $y = 4^x - 1$



4. $y = 0.5(2)^{x-1}$



cannot be a fraction

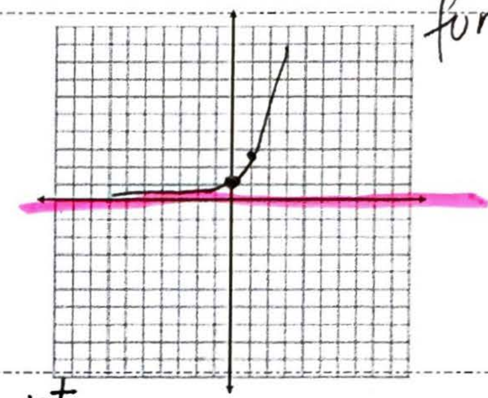
Key Concepts

e - an irrational number approximately equal to 2.71828... e is useful for describing continuous compounding functions

Examples

5. Graph $y = e^x$. Evaluate e^3 to four decimal places.

x	y
0	1
1	e



Key Concepts

Continuously Compounded Interest Formula $A = P e^{rt}$ (initial)

A = amount in account   P = principal   r = annual rate or interest/in decimal form   t = time in years

Real life problem- COMPARING COMPOUNDFORMULAS (yearly 1, semiannually 2, quarterly 4, monthly 12, daily 365, weekly 52 VS. continuous compounding)

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad y = a \cdot b^x$$

6. Suppose you invest \$100 at an annual interest rate of 4.8% compounded continuously. How much will you have in the account after 3 years? Compare the continuous compounding amount, to the amount compounded monthly. Which compounding saves you more money?

$P = 100$

$r = 4.8\% = \boxed{0.048}$

continuous

$A = P e^{rt}$

$t = 3$

$A = 100 \cdot e^{0.048 \cdot 3} = \underline{115.49}$ Higher

monthly $n = 12$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 100\left(1 + \frac{0.048}{12}\right)^{12 \cdot 3} = \underline{115.46}$

100%   extra

7. Archaeologists use carbon-14 with a half-life of 5730 years to determine the age of artifacts in carbon dating. Write an exponential decay function for a 24-mg sample. How much carbon -14 remains after 1000 years?

$t = 5730$ half life 12

$P = 24$

$t = 1000$

$12 = 24 \cdot b^{5730}$

Need logs!

Mini-white board activity- Page 193

Please write the question down and the explanation to how you knew what the answer is.

1)   *2)*   *3)*   *4)*

Challenge/Extra credit: Think about a plan page 190 or TEXTBOOK PAGE 421 TASK 1

Exit ticket: **Graph the function y= $2(4)^{x-1}$ -3. Use the GRAPH PAPER that's in your baskets!**

Standards: **A2. F.LE.A.2** (formerly F-LE.A.4). For exponential models, express as a logarithm the solution to $ab^c = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. **A2. F.IF.A.1** (formerly F-IF.B.4) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **A2.F.IF.A.2** (formerly F-IF.B.6)Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Objectives; Students will be able to write and evaluate logarithmic expressions from the corresponding (inverse) exponential form expressions and vice versa. Students will be able to graph logarithmic functions and compare them to the exponential functions (inverse).

Section 7.3: Logarithmic Functions as Inverses

Warm Up Using previous knowledge of exponents (integer and rational) solve the following x values:

1. $8 = x^3$ 2. $x^{1/4} = 2$ 3. $27 = 3^x$ 4. $4^{3x} = 2^6$ 5. $2^x = 1/16$ 6. $2^x=5$

Converting exponential form to logarithmic form and vice versa. $\boxed{b^x = a \leftrightarrow \log_b a = x}$

Examples

1. Write in logarithmic form. $\log_b 1 = 0$

a) $4^{-3} = \frac{1}{64}$ b) $6^2 = 36$ c) $17^0 = 1$

$\log_4 \frac{1}{64} = -3$ $\log_6 36 = 2$ $\log_{17} 1 = 0$

2. Write in exponential form.

a) $\log_2 8 = 3$ b) $\log_{10} 1000 = 3$ c) $\log_8 \frac{1}{4} = -\frac{2}{3}$

$2^3 = 8$ $10^3 = 1000$ $8^{-2/3} = \frac{1}{4}$

3.What is the inverse function of $y = 2^x$?

switch x and y $x = 2^y$

$y = \log_2 x$

4. What is the inverse function of $y = \log x$?

$x = \log_{10} y$ $y = 10^x$

5. Evaluate and conclude two "rules" from the following examples:

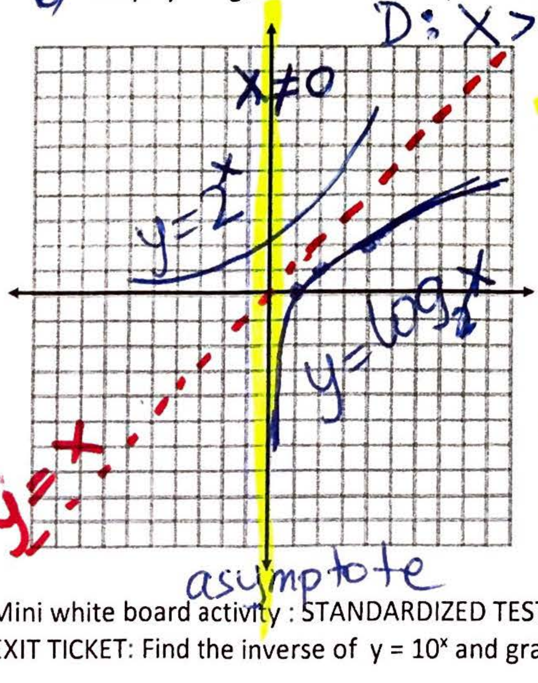
a) $\log_2 64 =$ b) $\log_8 2 =$ c) $\log 1000 =$ 3 d) $\log_2 2 =$ 1 e) $\log_{10} 10 =$ 1 f) $\log_{10} 0 =$ DNE

$\log_2 2^6 = 6$ $\boxed{\log_b b^a = a}$ $\log_b b = 1$ $\log - = \text{DNE}$

Logarithm is the inverse of an exponential function. This means the graphs are reflexive over $y=x$ line.

3) Graph $y = \log_2 x$ and its inverse, find domain and range.

D: $x>0$



x	y
0	DNE
1	0
2	1
4	2

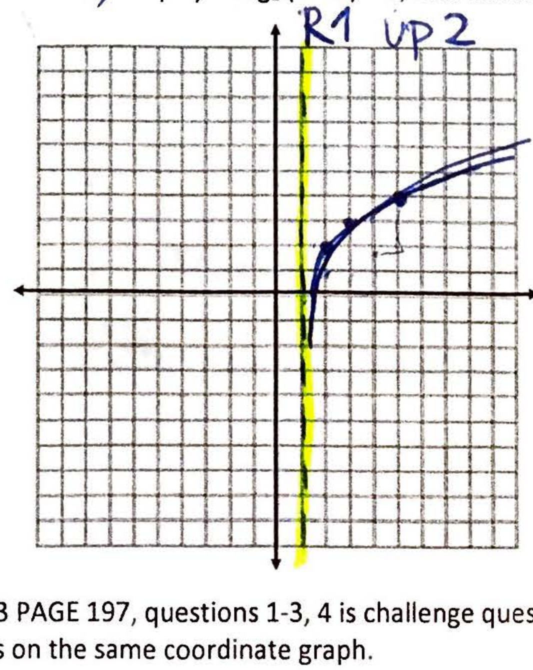
$\log_2 1 = 0$

$\log_2 2 = 1$

$\log_2 4 = 2$

7) Graph $y = \log_2 (x-1) + 2$, find domain and range.

R1 up2



D: $x>1$

Mini white board activity : STANDARDIZED TEST PREP LESSON 7-3 PAGE 197, questions 1-3, 4 is challenge question.

EXIT TICKET: Find the inverse of $y = 10^x$ and graph both functions on the same coordinate graph.

Standards: **A2. A.SSE.B.2** (formerly A-SSE.B.3c) Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Use the properties of exponents to rewrite expressions for exponential functions.
Objectives: Students will understand the relationship between properties of exponents and properties of logarithms and apply then to produce equivalent expressions in different forms.

Section 7.4: Properties of Logarithms

Warm Up Simplify, if possible, then evaluate each expression for $x = 2$.

1. x^3/x^1 2. $x^5 x^2$ 3. $(x^3)^2$

$\frac{x \cdot x \cdot \cancel{x}}{\cancel{x}}$ $= x^2 = 2^2 = 4$ $= x^7 = 2^7 = 128$ $x^3 \cdot x^3 = x^6$ $2^6 = 64$

Key Concepts
Properties of Logarithms

how does it connect to properties of exponents?

$\log_b MN = \log_b M + \log_b N$	Product Property	$x^m \cdot x^n = x^{m+n}$
$\log_b M/N = \log_b M - \log_b N$	Quotient Property	$x^m / x^n = x^{m-n}$
$\log_b M^n = n\log_b M$	Power Property	

$\log x^3 = 3\log x$

Examples

1. Write each logarithmic expression as a single logarithm, and write the name of property(ies) used, evaluate if possible.
 a. $\log_4 64 - \log_4 16$ b. $6 \log x + \log y$ c. $3\log_2 4 + \log_2 y - 2 \log_2 x$

quotient property
$\log_4 \frac{64}{16} = \log_4 4 = 1$

power P. $\log x^6 + \log y$
product P. $\log x^6 y$

$\log_2 4^3 + \log_2 y - \log_2 x^2$
$\log_2 \frac{64y}{x^2}$

2. Expand each logarithm, write the name of property(ies) used, evaluate if possible.
 a. $\log_7 (7/u)$ b. $\log(4p^3)$ c. $\log_2(4p)^3$

Quotient
$\log_7 7 - \log_7 u$
$1 - \log_7 u$

just P
$\log 4 + \log p^3$ product
$\log 4 + 3\log p$ power

both 4 and P
$\log_2 4^3 + \log_2 p^3$
$= 3\log_2 4 + 3\log_2 p$

Key concept- Change of Base Formula/property: $\log_c M = \log_b M/\log_b C$, the base b is usually 10, unless otherwise noted. *Note: A log with any base can be entered on newer versions of calculators, however, if not, this formula has to be used when evaluating logarithms with bases different than 10 and e (log and ln).*

3. Use the Change of Base Formula to rewrite $\log_6 12$ then solve using your calculator.

$\log \frac{12}{6} = \log 2$ ✓

$\frac{\log 12}{\log 6} \neq \log 2$!!!
≈ 1.39

~~Word problems: Page 197 questions 4 and 5, page 201 question 5.~~

Pg 197 1-3
Pg 201 1-4

Mini white board activity: STANDARDIZED TEST PREP LESSON 7-4 PAGE 201, questions 1-4
Challenge/Early finishers: think about a plan worksheet lesson 7-4 page 198
Exit ticket: Expand log $(x/y)^3$ and write the names of properties you used.

4th

Standards; **A2. F.IF.B.4.** (formerly F-IF.C.8b) Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the properties of exponents to interpret expressions for exponential functions. **A2. N.Q.A.1** (formerly N-Q.B.2) Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling. **A2.F.LE.A.2** (formerly F-LE.A.4) For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Objectives: Students will be able to solve exponential and logarithmic equations with base 2, 10 and e.

Section 7-5/ 7-6: Solving Exponential and Logarithmic Equations/Natural Logarithms

Warm Up

Write each expression as a single logarithm.

1. $\ln 12 - \ln 3$
2. $3\log_{11}5 + \log_{11}7$

Expand each logarithm

3. $\log_c(a/b)$
4. $\log_3 x^4$
5. $\ln(3x/2)$

*To solve exponential equations with **e** as a base, use **ln** to "cancel out" e. To cancel out other bases use the log with the base the same as the base given in the question.*

Examples of exponential equations

1. Solve $2^{2x} = 16$.

Use log with base 2

$\cancel{\log_2} 2^{2x} = \log_2 16 \qquad 2^4 = 16$

$\frac{2x}{2} = \frac{4}{2}$ $\boxed{x=2}$

2. Solve $7 - 10^{2x-1} = 4$

Isolate the base

$-7 \qquad -7$

$\cancel{\log 10}^{2x-1} = \log 3$

$2x - \cancel{1} = \log 3 + 1$

$+1$

$x = \frac{\log 3 + 1}{2}$

3. Solve $3 + e^{3x+2} = 20$

Insert ln on both sides

$-3 \qquad -3$

$\cancel{\ln e}^{3x+2} = \ln 17$

$3x + \cancel{2} = \ln 17 - 2$

-2

$\frac{3x}{3} = \frac{\ln 17 - 2}{3}$

To solve logarithmic equations, write the problem in exponential form and solve

Examples of logarithmic equations

Lesson 7-3

4. Solve $\log(2x-2) = 4$

equals

10 to the power

$10^4 = 2x - 2$

$10{,}000 = 2x - \cancel{2}$

$+2 \qquad +2$

$\frac{10{,}002}{2} = \frac{2x}{2}$ $\boxed{x = 5001}$

5. Solve $3\log x - \log 2 = 5$.

$\log x^3 - \log 2 = 5$

$\log \frac{x^3}{2} = 5$ ✓

$2 \cdot 10^5 = \frac{x^3}{\cancel{2}} \cdot \cancel{2}$

$x = \sqrt[3]{2 \cdot 10^5} \approx 58.48$

6. Solve $3\ln x + 2\ln 2 = 8$ $= \ln x^3 + \ln 2^2$

$\ln_e x^3 \cdot 4 = 8$

$\frac{e^8}{4} = \frac{x^3 \cdot \cancel{4}}{\cancel{4}}$

$x = \sqrt[3]{\frac{e^8}{4}} \approx 9.07$

Mini white board activity: STANDARDIZED TEST PREP LESSON 7-5 PAGE 205, questions 1-4, page 209 questions 1-4

pg 205 2, 3

Challenge/Early finishers: think about a plan worksheet lessons 7-5 and 7-6 (page 202 and 206)

Exit ticket: Solve: 1) $2^{3x+1} = 7$ 2) $\log(2x+4) = 2$

Standards; **A2. F.IF.B.4**. (formerly F-IF.C.8b) Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the properties of exponents to interpret expressions for exponential functions. **A2. N.Q.A.1** (formerly N-Q.B.2) Identify, interpret, and justify appropriate quantities for the purpose of descriptive modeling. **A2.F.LE.A.2** (formerly F-LE.A.4) For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Objectives: Students will be able to solve exponential and logarithmic equations with base 2, 10 and e and apply them to real life problems.

Section 7-5/ 7-6: Solving word problems (exponential and logarithmic)

Warm Up

Write the expression as a single logarithm.

1. $\log_5 y - 4(\log_5 r + 2\log_5 t)$

Expand the logarithm

2. $\log 7(3x-2)^2$

Examples

1. By measuring the amount of carbon-14 in an object, a paleontologist can determine its approximate age. The amount of carbon-14 in an object is given by $\boxed{y = ae^{-0.00012t}}$, where *a* is the amount of carbon-14 originally in the object, and *t* is the age of the object in years.

AMOUNT

A fossil of a bone contains 83% of its original carbon-14. What is the approximate age of the bone? $t = ?$

$y = ae^{-0.00012t}$

$\frac{83}{100} = \frac{\cancel{100}}{100} e^{-0.00012t}$ ← number ≈ 2.78

$\frac{\ln 0.83}{-0.00012} = \frac{\cancel{\ln e}^{-0.00012t}}{-0.00012}$

$\boxed{t = 1553}$

2. The formula for the maximum velocity *v* of a rocket is $\boxed{v = -0.0098t + c \ln R}$, where *c* is the exhaust velocity in km/s, *t* is the firing time, and *R* is the mass ratio of the rocket. A rocket must reach 7.7 km/s to attain a stable orbit 300 km above Earth.

MAX VELocity $V = -0.0098t + c \ln R$ ← MASS RATIO; FIRING TIME; exhaust vEl.

What is the maximum velocity of a rocket with a mass ratio of 18, an exhaust velocity of 2.2 km/s, and a firing time of 25 s?

$V = ?$ $R = 18$ $c = 2.2$ $t = 25$

$V = -0.0098 \cdot 25 + 2.2 \ln 18$

$V = 6.11 \frac{km}{s}$ this velocity is not enough for a stable orbit

3. **Suppose you deposit $2500 in a savings account that pays you 5% interest per year.**

a. How many years will it take for you to double your money?

b. How many years will it take for your account to reach $8,000?

$r = +5\%$ $P = 2500$

a) double your money = 5000

$A = P(1+r)^t$ $y = ab^x$

100% GROWth factor $r = 5\% = 0.05$

$\frac{5000}{2500} = \frac{25\cancel{0}0}{2500}(1.05)^t$

$\log 2 = \log 1.05^t$

$\frac{\log 2}{\log 1.05} = \frac{t \cancel{\log 1.05}}{\cancel{\log 1.05}}$ $t = 14.2 \approx 14$

4. The equation $\boxed{y = 281(1.01)^x}$ is a model for the population of the United States *y*, in millions of people, *x* years after the year 2000. Estimate when the United States population will reach 400 million people. $t = ? = x$

$y = 400$

$y = 281(1.01)^x$

$\frac{400}{281} = \frac{\cancel{281}}{\cancel{281}}(1.01)^x$

$\log_{1.01} \frac{400}{281} = \cancel{\log_{1.01}(1.01)}^t$

$t = \log_{1.01} \frac{400}{281} \approx 35.5$

in 2035

Mini white board activity: STANDARDIZED TEST PREP LESSON 7-5 PAGE 205, question 5, page 209 question 5

Challenge/Early finishers: think about a plan worksheet lessons 7-5 and 7-6 (page 202 and 206)