$\sqrt{4^2} = \sqrt{4} \cdot \sqrt{4} = 4$

Standards: A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

Objective: Students can use the conjugate root theorem and Decartes' Rule of Signs to determine the number of roots and to create polynomials.

$i = \sqrt{-1}$ $\quad i^2 = -1$

Section 5-5 (Part 2) Theorems About Roots of Polynomial Equations

CONJUGATE ROOT THEOREM

Warm Up

Multiply. $\quad (a+b)(a-b) = a^2 - b^2$

1. (-4i)(6i)

$-24i^2$

$-24(-1) = \boxed{24}$

2. (2 + i)(2 - i)

$= 2^2 - i^2$

$= 4 - (-1)$

$= 4 + 1 = \boxed{5}$

3. (1 + √3)(1 - √3)

$= 1^2 - \cancel{\sqrt{3}}^{\,2}$

$= 1 - 3 = \boxed{-2}$

Key Concepts

Conjugate Root Theorem
If P(x) is a polynomial with rational coefficients, then the irrational roots of P(x) occur in pairs.
If a - √b is an irrational root, then a + √b is also a root. $\quad a - \sqrt{b},\ a + \sqrt{b}$ irrational

Imaginary Root Theorem
If the imaginary number a + bi is a root of a polynomial equation with real coefficients, then the conjugate a – bi also is a root. $\quad a + bi,\ a - bi$ imaginary

Examples

1. A cubic polynomial P(x) has real coefficients. If 2 - 3i and $\frac{3}{4}$ are two roots of P(x). Are there any additional roots?

degree 3 (3 Linear factors) $\quad \frac{3}{4}$ RATIONAL $\quad 2 - 3i$ imaginary $\xrightarrow{\text{conjugate}}$ $2 + 3i$ yes

2. A quartic polynomial P(x) has real coefficients. If √3 and 6 - 2i are two roots of P(x), what are the other two roots? Create a polynomial with such roots.

degree 4 (4 Linear f.)

$a + \sqrt{b}$

$0 + \sqrt{3}$ irrational, $-\sqrt{3}$, $6 - 2i$, $6 + 2i$

$(x - \sqrt{3})(x + \sqrt{3})(x - 6 + 2i)(x - 6 - 2i)$

$(x^2 - \cancel{\sqrt{3}}^{\,2})((x-6)^2 - (2i)^2)$

$(x^2 - 3)(x^2 - 12x + 36 - 4(-1))$

	x^2	-3
x^2	x^4	$-3x^2$
$-12x$	$-12x^3$	$36x$
40	$40x^2$	-120

$x^4 - 12x^3 + 37x^2 + 36x - 120$

3. Find a third degree polynomial with rational coefficients that has roots -2, and 2 - i.

$-2, \quad 2 - i \xrightarrow{\text{conjugate pair}} 2 + i$

$(x + 2)(x - 2 + i)(x - 2 - i)$

$(x + 2)((x - 2)^2 - i^2)$

$(x + 2)(x^2 - 4x + 4 - (-1))$

	x	$+2$
x^2	x^3	$2x^2$
$-4x$	$-4x^2$	$-8x$
5	$5x$	10

$x^3 - 2x^2 - 3x + 10$

Classwork: WORKBOOK page 137 questions 1,2,3,5, 6 (separate sheet of paper/ show work!!!)

Extra credit: Think about a Plan workbook page 134

Exit ticket: Create a quadratic function that has one root $1 + \sqrt{5}$.

1ST PERIOD

Standards: A2.A.APR.A.1 (formerly A-APR.A.2) Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

Objective: Students will use Rational Root Theorem in order to determine the possible rational roots, then test those values using the knowledge of Remainder Theorem and apply them to real life problems.

5-5 (Part 1): Theorems about roots of polynomial equations
RATIONAL ROOT THEOREM

Warm Up

1. What is P(-3) for the polynomial $P(x)= -x^3+4x^2-5x -3$? Is -3 a zero of this polynomial? How do you know?

$$\begin{array}{r|rrrr} -3 & -1 & 4 & -5 & -3 \\ & \downarrow & 3 & -21 & 78 \\ \hline & -1 & 7 & -26 & 75 \end{array} \; R$$

NO, because $R \neq 0$

$P(-3) = 75$

2. What are all the factors of number 12? 1, 2, 3, 4, 6, 12

3. What is a definition of RATIONAL numbers? If a number is not rational, what else can it be?

number can be written as a fraction, decimals have a pattern

irrational: π, $\sqrt{\text{prime}}$, imaginary (complex

Key Concepts

Rational Root Theorem

Let $P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ be a polynomial with integer coefficients. Then there are a limited number of **possible** RATIONAL roots of P(x) = 0:

- Integer roots must be factors of a_0. — constant term
- Rational roots must have reduced form p/q where p is an integer factor of a_0 and q is an integer factor of a_n. ← leading coefficient

Steps to Finding Rational Roots

1. Find all factors of the constant term (p), find all factors of the leading coefficient (q)
2. Find all possible "combinations" of factors of p divided by factors of q, make all numbers + or -

Examples

1. Find the possible rational roots of $1x^3 - 2x^2 + 2x - 4 = 0$ then test the values using synthetic (or long division) to determine if the values are the actual zeros of this equation. Then determine the remaining (irrational or imaginary) zeros of the equation.

p = −4 ; 1, 2, 4

q = 1 ; 1

possible roots: ±1, ±2, ±4

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 2 & -4 \\ & \downarrow & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -3 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & 2 & -4 \\ & \downarrow & -1 & 3 & -5 \\ \hline & 1 & -3 & 5 & -9 \end{array}$$

$P(4) = 44$

$P(2) = 0$

$P(-2) = -24$

$P(-4) = -108$

$$\begin{array}{r|rrrr} -4 & 1 & -2 & 2 & -4 \\ & \downarrow & -4 & 24 & -104 \\ \hline & 1 & -6 & 26 & -108 \end{array}$$

$x^2 + 2 = 0$

$x^2 = -2$ $\boxed{x = \pm\sqrt{2}i}$

2. Find the rational roots of $3x^3 - x^2 - 15x + 5 = 0$ then test the values using synthetic (or long division) to determine if the value is a zero of this equation. Then determine the remaining (irrational or imaginary) zeros.

p = 5 ; 1, 5

q = 3 ; ~~1~~, 3

possible roots: $\pm1, \pm5, \pm\frac{1}{3}, \pm\frac{5}{3}$

$P(1) =$ $P(5) = 280$ $P(\frac{1}{3}) = 0$ $P(\frac{5}{3}) =$

$P(-1) =$ $P(-5) =$ $P(-\frac{1}{3}) =$ $P(-\frac{5}{3}) =$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & -15 & 5 \\ & \downarrow & 1 & 0 & -5 \\ \hline & 3 & 0 & -15 & 0 \end{array}$$

↑ R

$3x^2 - 15 = 0$

$3x^2 = 15$

$x^2 = 5$

$\boxed{x = \pm\sqrt{5}}$

Classwork: TEXTBOOK page 315 questions 1,2,3

Extra credit: Think about a Plan workbook page 134

Exit ticket: FIND ALL RATIONAL ROOTS OF $15x^3-32x^2+3x+2=0$