1st period

Standards: A2.F.IF.A.1 (formerly F-IF.B.4) For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. ★
Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.
Objective: Students can use the conjugate root theorem and Decartes' Rule of Signs to determine the number of roots and to create polynomials.

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

Section 5-5 (Part 3) Theorems About Roots of Polynomial Equations

DECARTES' RULE OF SIGNS

Warm up

Create a cubic polynomial with one root of -1 (RATIONAL), and the second root of 2-3i (COMPLEX (imaginary)).

$-1,\ 2-3i,\ 2+3i$ (conjugate)

$(x+1)(x-2+3i)(x-2-3i)$

$(x+1)((x-2)^2-(3i)^2)$

$(x+1)(x^2-4x+4-9(-1))$

	x^2	$-4x$	$+13$
x	x^3	$-4x^2$	$13x$
$+1$	x^2	$-4x$	13

$\boxed{x^3-3x^2+9x+13}$

Key Concepts

> **Decartes' Rule of Signs** number of positive and negative Real ROOTS
> Let P(x) be a polynomial with real coefficients written in standard form.
> *The number of positive real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(x) or is less than that by an even number.
> *The number of negative real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(-x) or is less than that by an even number.

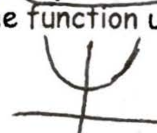
Examples

1. What does Decartes' Rule of Signs tell you about the real roots of $x^3 - x^2 + 1 = 0$? Sketch the function using your calculators and circle the actual number of real roots.

Positive Roots:
$P(x) = +x^3 - x^2 + 1$
2 or 0 (NONE)

negative ROOTS: — even power
$P(-x) = (-x)^3 - (-x)^2 + 1$
$-x^3 - x^2 + 1$
1 sign change
1 NEG. ROOT

2. What does Decartes' Rule of Signs tell you about the real roots of $2x^2 + 5 = 0$? Sketch the function using your calculators and circle the actual number of real roots.



\# POSITIVE ROOTS:
$P(x) = +2x^2 + 5$
0 positive ROOTS (NO SIGN changes)

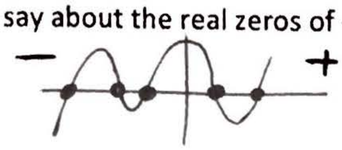
\# NEG. ROOTS:
$P(-x) = 2(-x)^2 + 5$
$P(-x) = +2x^2 + 5$
0 Neg ROOT (NO sign changes)

3. Find the number of real roots of the polynomial using Descartes' Rule of Signs. Sketch the function using your calculators and circle the actual number of real roots. $P(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

\# positive ROOTS:
$P(x) = +x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$
2 or 0 positive Real roots

\# NEG. ROOTS:
$P(-x) = (-x)^5 + 3(-x)^4 - 5(-x)^3 - 15(-x)^2 + 4(-x) + 12$
$P(-x) = -x^5 + 3x^4 + 5x^3 - 15x^2 - 4x + 12$
3 or 1 negative Real roots

Classwork: WORKBOOK page 137 questions 4
Extra credit: Think about a Plan workbook page 134
Exit ticket: What does Decartes' rule of sign say about the real zeros of $-x^3 + 5x^2 - 2x - 1$



tandard: A2.F.IF.B.3 (formerly F-IF.C.7c) Graph functions expressed symbolically and show key features of the graph, by hand in simple ases and using technology. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end ehavior.

bjective: Students will model real life problems using polynomials. Students will compare models and determine the best degree olynomial given some ordered pairs.

5-8 Polynomial Models in the Real World

Varm up

Determine the number of possible positive and negative real zeros using Decartes' rule of sign

$f(x) = -x^3 + 5x^2 - 7x - 3$

1 negative root!

# of positive real roots

$f(x) = -x^3 + 5x^2 - 7x - 3$

2 or 0 positive roots

# of negative real roots:

$f(-x) = -(-x)^3 + 5(-x)^2 - 7(-x) - 3$

$f(-x) = +x^3 + 5x^2 + 7x - 3$

Key Concept

polynomial models -for any $n + 1$ points in the coordinate plane that pass the vertical line test, there is a unique polynomial of degree at most n that fits the points **perfectly**.

X does not repeat

Examples 2 pts linear d1 3pt quadratic

1. What polynomial function has a graph that passes through the four points (0, -3), (1, -1), (2, 5) and (-1, -7)?

L1 (x values) L2 (y-values) 4-1=3 cubic function

L1	L2
0	-3
1	-1
2	5
-1	-7

a = 1
b = -1
c = 2
d = -3

$y = 1x^3 - 1x^2 + 2x - 3$ (a, b, c, d)

2. Find a polynomial whose graph passes through (-2, -16), (3, 11) and (0, 2).

quadratic 3 - 1 = 2

a = -1.2
b = 6.6
c = 2

$y = -1.2x^2 + 6.6x + 2$ (a, b, c)

3. For the data below that examines U.S. Federal Spending, compare two models and determine which one best fits the data. Which model seems more likely to represent the data set over time?

Year		Total (billions)
1965	0	630
1980	15	1300
1995	30	1950
2005	40	2650

4 points try cubic

cubic
a = 0.03
b = -1.29
c = 57.83
d = 630
$\boxed{r^2 = 1}$

quadratic
a = 0.37
b = 34.51
c = 646.40
$\boxed{r^2 = 0.996}$

better model

Classwork: WORKBOOK page 149 (questions 1-4)

Extra credit: Think about a Plan workbook page 146

Exit ticket: workbook page 149 question 5