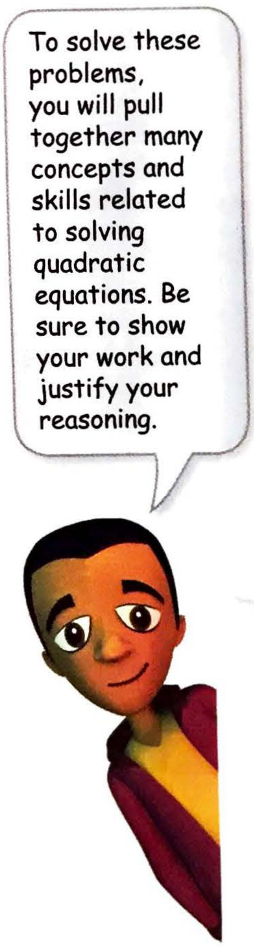
4 Pull It All Together



BIG idea Equivalence and Function

The parameters a, b, c, h, and k in the standard and vertex forms of a quadratic function give information on how the graph of the function relates to the graph of the parent function $y = x^2$.

Standard form: $y = ax^2 + bx + c$ Vertex form: $y = a(x - h)^2 + k$

Task 1

Refer to the two forms shown above.

a. What information do the parameters, or combinations of parameters, provide about the graph of the quadratic function?

b. Begin with standard form. Transform it to vertex form. What are the values of h and k in terms of a, b, and c?

c. Show how the Quadratic Formula follows from your result in part (b). *Hint:* Set the expression in your vertex form equal to 0. Then solve by factoring.

BIG idea Solving Equations and Inequalities

A problem may require different types of equation solving. You should know when and how to use a graphing calculator to help you with your work.

Task 2

You shoot an arrow at a target. The parabolic path of your arrow passes through the points shown in the table.

x	y
30	6
60	7
100	4

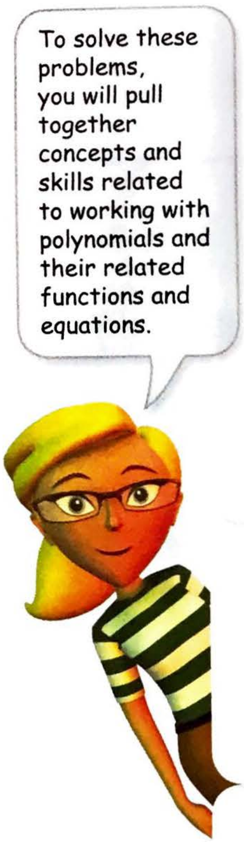
a. Find a quadratic function in standard form that models the path of your arrow. *Hint:* The three points are (x, y)-values that satisfy $y = ax^2 + bx + c$.

b. If the y-value represents height above the ground, for what value of x would your arrow hit the ground if you miss the target?

c. If the target bull's-eye is at $x = 100$, at what height should the bull's-eye be for your arrow to hit it?

d. If the target bull's-eye is at height $y = 2.98$, at what value of x should the bull's-eye be for the arrow to hit it?

5 Pull It All Together



BIG idea Function

You can represent quantities using variables and algebraic expressions. You can represent some relationships between quantities using equations.

Task 1

The polynomial $2x^3 + 9x^2 + 4x - 15$ represents the volume in cubic feet of a rectangular holding tank at a fish hatchery. The depth of the tank is $(x - 1)$ feet. The length is 13 feet.

a. Use synthetic division to help you factor the volume polynomial. How many linear factors should you look for? What are they?

b. Assume the length is the greatest dimension. Which linear factor represents the 13-ft length? What are the dimensions of the tank? What is its volume? What is the value of x? Do you get the same volume if you substitute the value of x into $2x^3 + 9x^2 + 4x - 15$?

BIG idea Equivalence

You can use the Binomial Theorem and properties of algebra to rewrite some powers.

Task 2

Show that the following equation is true for all values of a and b.

$$[(a - b) + 1]^5 = a^5 - 5a^4(b - 1) + 10a^3(b - 1)^2 - 10a^2(b - 1)^3 + 5a(b - 1)^4 - (b - 1)^5$$

BIG idea Solving Equations and Inequalities

A polynomial $P(x)$ of degree n, $n \geq 1$, and its related polynomial function $y = P(x)$ have n complex zeros. The zeros are identical to the n complex roots of the related polynomial equation $P(x) = 0$.

Task 3

Four of these five polynomial functions have identical zeros. The fifth has exactly two zeros in common with each of the other functions. Write this fifth function as a product of its linear factors.

$$P_1(x) = x^3 - 6x^2 + 11x - 6$$
$$P_2(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$$
$$P_3(x) = x^4 - 7x^3 + 17x^2 - 17x + 6$$
$$P_4(x) = x^4 - 8x^3 + 23x^2 - 28x + 12$$
$$P_5(x) = x^4 - 9x^3 + 29x^2 - 39x + 18$$

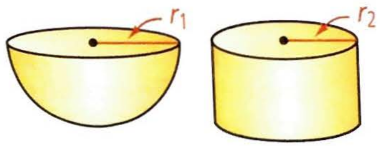
6 Pull It All Together

BIG idea Solving Equations and Inequalities

Solving an equation is the process of rewriting the equation to make what it says about its variables as simple as possible.

Task 1

An environmental equipment supplier sells hemispherical holding ponds for treatment of chemical waste. The volume of a pond is $V_1 = \frac{1}{2}\left(\frac{4}{3}\pi r_1^{\,3}\right)$, where r_1 is the radius in feet. The supplier also sells cylindrical collecting tanks. A collecting tank fills completely and then drains completely to fill the empty pond. The volume of the tank is $V_2 = 12\pi r_2^{\,2}$, where r_2 is the radius of the tank.



a. Since $V_1 = V_2$, write an equation that shows r_1 as a function of r_2. Write an equation that shows r_2 as a function of r_1.

b. You want to double the radius of the pond. How will the radius of the tank change?

BIG idea Solving Equations and Inequalities

The numbers and types of solutions vary based on the type of equation.

BIG idea Function

You can represent functions in a variety of ways (such as graphs, tables, equations, or words). Each representation is particularly useful in certain situations.

Task 2

Suppose $f(x) = \sqrt{x + 1}$.

a. What are the domain and range of f?

b. Find $f^{-1}(x)$. What are its domain and range? Be careful!

c. Show that $(f \circ f^{-1})(a) = a = (f^{-1} \circ f)(a)$ for any a in the respective domains.

d. Solve the equation $f(x) = f^{-1}(x)$. Remember to check for extraneous roots.

e. Graph the functions f and f^{-1}. Be sure that you accurately represent the domains of each function. Interpret graphically the solution(s) you found to the equation in part (d).

7 Pull It All Together

BIG idea Modeling

You can represent many real-world mathematical problems algebraically. An algebraic model can lead to an algebraic solution.

Task 1

Suppose you invest a dollars to earn an annual interest rate of r percent (as a decimal). After t years, the value of the investment with interest compounded yearly is $A(t) = a(1 + r)^t$. The value with interest compounded continuously is $A(t) = a \cdot e^{rt}$.

a. Explain why you can call $e^r - 1$ the effective annual interest rate for the continuous compounding.

b. Suppose you can earn interest at some rate between 0% and 5%. Use your knowledge of the exponential function to explain why continuous compounding does not give you much of an investment advantage.

c. For each situation find the unknown quantity, such that continuous compounding gives you a \$1 advantage over annually compounded interest.

- How much must you invest for 1 year at 2%?
- At what interest rate must you invest \$1000 for 1 year?
- For how long must you invest \$1000 at 2%?

BIG idea Function

You can use transformations such as translations, reflections, and dilations to understand relationships within a family of functions.

Task 2

$f(x) = b^x$ and $g(x) = \log_b x$ are inverse functions. Explain why each of the following is true.

a. The translation $f_1(x) = b^{x-h}$ of f is equivalent to a vertical stretch or compression of f.

b. The inverse of $f_1(x) = b^{x-h}$ is equivalent to a translation of g.

c. The inverse of $f_1(x) = b^{x-h}$ is not equivalent to a vertical stretch or compression of g.

d. The function $h(x) = \log_c x$ is a vertical stretch or compression of g or of its reflection $-g$.