Modeling a Quadratic Function When Given a Graph
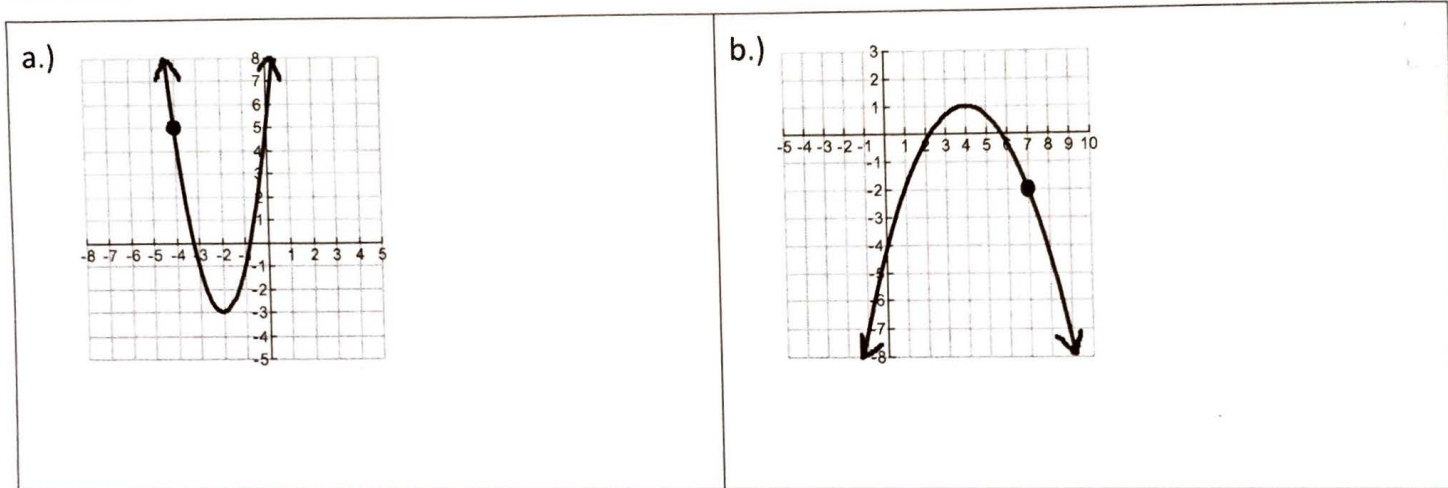
Example 1: Write a quadratic function (in vertex form) that models each graph.

a.)

b.)

Modeling a Quadratic Function Using Various Word Problems

Example 2: Complete each word problem using techniques learned in previous concepts.

a.) Courtney is building a rectangular wading pool. She wants the area of the bottom to be 54 ft². She also wants the length of the pool to be 3 ft longer than twice its width.
What are the dimensions of the pool?

b.) The formula for throwing a baseball in the air is represented by $h = -16t^2 + 12t + 40$ where h is the height of the ball. After how many seconds will the ball hit the ground?

c.) The function $h = -16t^2 + 1700$ gives an object's height h, in feet, at t seconds.

i.) What does the constant tell you about the height of the object?

ii.) What does the coefficient of t^2 tell you about the direction the object is moving?

iii.) When will the object be 1000 feet above the ground?

iv.) What are a reasonable domain and range for the function h?

Example 2 Cont'd: Complete each word problem using techniques learned in previous concepts.

d.) The equation $y = x^2 - 12x + 45$ models the number of books y sold in a bookstore x days after an award-winning author appeared at an autograph-signing reception. What was the first day that at least 100 copies of the book were sold?

e.) A ball is thrown into the air with an initial upward velocity of 48 ft/s. It height h in feet after t seconds is given by the function $h(t) = -16t^2 + 48t + 4$.

i.) What height will the ball be when 2 seconds has passed?

ii.) In how many seconds will the ball reach its maximum height?

iii.) What is the ball's maximum height?

③ Anne's rectangular flower garden measures 20 m by 30 m in size. She plans on doubling the flower garden's area by adding a strip of uniform width around it. Determine the width of the strip around the flower garden.

④ A mural is to be painted on a wall that is 15 m long and 12 m high. A border of uniform width is to surround the mural. If the mural is to cover 75% of the area of the wall, how wide must the border be?

Name: _____   Date: _____

Solving Quadratic Word Problems I

Quadratic equations arise naturally when one solves problems from a variety of contexts, including area, motion, economics, and growth rates of populations. In fact, any problem situation in which one quantity depends upon the product of two linear quantities yields an analysis of a quadratic equation. Over the next two lessons we will solve classic quadratic word problems. As always, read each question thoroughly to understand what is given and what is being asked.

Exercise #1: The product of two consecutive even integers is 48. Find all sets of these integers that satisfy this description using both guess-and-check and algebraic methods.

(a)   Guess-and-Check Approach   **(b)** Algebraic Approach

n	$n+2$	$n(n+2)$

One of the severe limitations in a guess-and-check approach is that it often misses answers. For each of the remaining exercises, solve exclusively using **algebraic techniques**.

Exercise #2: Find three consecutive positive integers such that the product of the first two is 22 less than 11 times the third.

Exercise #3: The product of two consecutive odd integers is equal to 30 more than the first. Find the integers.

Exercise #4: An object is moving in a straight line. It initially travels at a speed of 9 meters per second, and it speeds up at a constant rate of 2 meters per second each second. Under such conditions, the distance d, in *meters*, that the object travels is given by the equation $d = t^2 + 9t$, where t is in seconds. According to this equation, how long will it take the object to travel 22 meters?

Exercise #5: A rectangular picture has a width that is two-thirds its length. The picture has an area of 294 square inches. What are the dimensions of the picture?

Exercise #6: A square is altered so that one dimension is increased by 4, while the other dimension is decreased by 2. The area of the resulting rectangle is 55. Find the area of the original square.

Exercise #7: The profit P, in *dollars*, gained by selling x computers is modeled by the equation $P = -5x^2 + 1000x + 5000$. How many computers must be sold to obtain a profit of $55,000.00?

QUADRATIC WORD PROBLEMS
HOMEWORK

Name: _____   Date: _____

Solving Quadratic Word Problems I

Applications

Solve each of the following quadratic word problems *algebraically*.

1. Two consecutive odd integers have a product of 99. Find all sets of integers that satisfy this description.

2. The product of two consecutive positive even integers is 14 more than their sum. Find the integers.

3. Find three consecutive positive integers such that the product of the first and the third is 29 more than the second.

4. The length of a rectangle is 4 less than twice the width. The area of the rectangle is 70. Find the dimensions of the rectangle.

5. An object is launched straight up into the air at an initial velocity of 64 feet per second. It is launched from a height of 6 feet off the ground. Its height H, in *feet*, at t seconds is given by the equation $H = -16t^2 + 64t + 6$. Find all times t that the object is at a height of 54 feet off the ground.

6. A rectangular picture has a height that is $\frac{5}{7}$ of its width. Its area is 140 square inches. What are the dimensions of the picture?

7. The square of a number decreased by 3 times the number is 28. Find all possible values for the number.

8. In a right triangle, the length of the longer leg is 7 more inches than the shorter leg. The length of the hypotenuse is 8 more inches than the length of the shorter leg.

   (a) If the shortest leg is represented by x, write expressions for the longer leg and the hypotenuse in terms of x. Label them on the diagram.

   (b) Write an equation using the Pythagorean Theorem that relates the three sides together and solve it for the value of x.

   (c) Find all three side lengths, and check your answer by verifying that $a^2 + b^2 = c^2$.
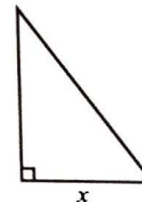
1. The graphs of $f(x) = x^2$ and $g(x) = x + 2$ are shown. Which statement explains the reason (2, 4) is a solution?

   A At (2, 4) the functions both have x- and y- values.

   B The domain and range of $f(x)$ and $g(x)$ are the same.

   C $x^2 = x + 2$ when $x = 2$

   D $f(x)$ and $g(x)$ intersect in the first quadrant.
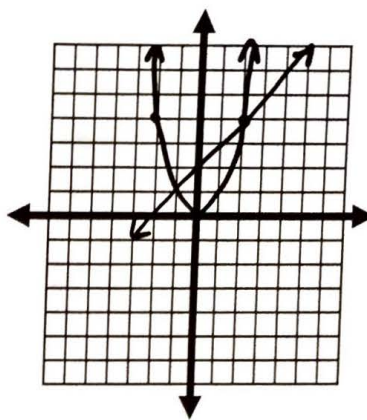
2. NO CALCULATOR! Fannie is making a rectangular blanket. The length of the blanket is 10 inches greater than its width, w, in inches. Write the function, $f(w)$, that describes the area, in square inches of Fannie's blanket as a function of the width.

3. Graph the function $g(x) = x^2 - 10x + 24$ on the coordinate plane. Be sure to label your x-intercept(s), y-intercept(s) and vertex.
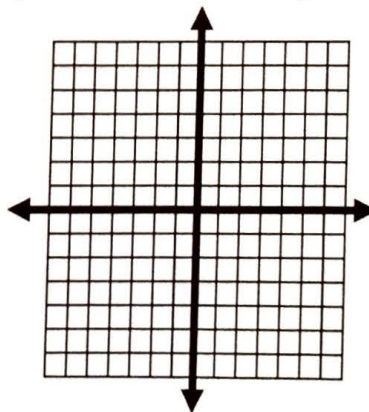
4. The length of a garden is 6 feet more than the width. The area is 40 square feet. Write an algebraic equation to determine the length and width of the garden. What is the length, in feet of the garden?

5. (Multiple Choice) The function $f(x)$ is given by the equation $f(x) = 3(x^2 + 4)$. The values for the quadratic function $h(x)$ are shown in the table. Which statement about the y-intercept of $f(x)$ is true?

   A. It is 10 units above the y-intercept of $h(x)$.
   B. It is 18 units above the y-intercept of $h(x)$.
   C. It is 12 units above the y-intercept of $h(x)$.
   D. It is 8 units above the y-intercept of $h(x)$.

x	-2	-1	0	1	2
$h(x)$	6	-3	-6	-3	6

6. Given $x^2 - y^2 = (x + y)(x - y)$. Correctly complete the sentence. To determine the value of $(42)(38)$ using the given equation, let x equal _____ and y equal _____.

7. A quadratic equation is given as $2x^2 + 4x + 8 = 0$. Write the solution(s) of the equation in equivalent form.

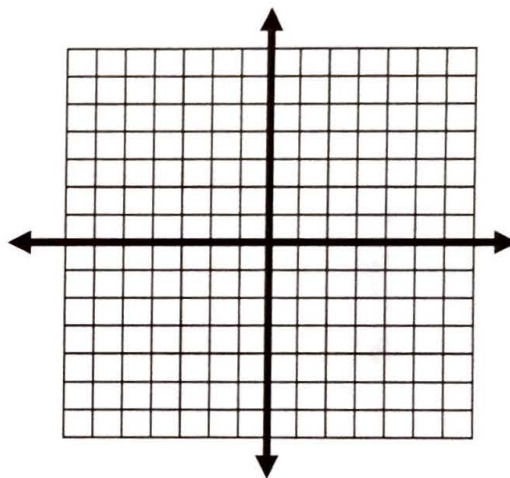8. Consider the system of equations.
$f(x) = x^2 + 6x - 4$
$g(x) = -|x - 3| - 1$
Graph the system.
What are **all** the values of x for which $f(x) = g(x)$?

10. Given the polynomial $2x^2 - x - 15$ determine the values of x where the graph of the polynomial crosses the x-axis.

11. The graph of a function $f(x) = -2x^2 + 3x + 5$ is a parabola. Plot the points for the x-intercept(s), y-intercept(s), and maximum or minimum point to the nearest half unit, whichever exists, on the coordinate plane provided.
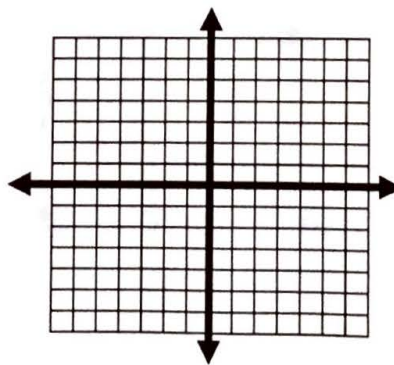
12. Solve the equation $2(x + 4)^2 - 113 = 49$

13. Solve the equation $-3x^2 + 2x = 8$

Solve the following <u>WORD PROBLEMS:</u> RE-DO SHOW WORK!

1. The length of a rectangle is 2 times its width. The area of the rectangle is 72 square inches. Find the dimensions of the rectangle.

2. The length of a rectangle is 4 times its width. The area of the rectangle is 144 square inches. Find the dimensions of the rectangle.

3. The length of a rectangular garden is 4 yards more than its width. The area of the garden is 60 square yards. Find the dimensions of the garden.

4. The width of a rectangle is 11 inches less than its length. Find the dimensions of the rectangle if he area is 80 square inches.

5. The length of a rectangle exceeds its width by 3 inches. The area of the rectangle is 70 square inches, find its dimensions.

6. The length of a rectangle is 3 centimeters more than the width. The area is 108 square centimeters. Find the length and width of the rectangle.

7. The width of a rectangle is 5 meters less than its length. The area is 84 square meters. Find the dimensions of the rectangle.

8. The length of a rectangle is twice the width. The area is 50 square inches. Find the dimensions of the rectangle.

9. The length of a rectangle is 1 foot more than twice the width. The area is 55 square feet. Find the dimensions of the rectangle.

10. The length of a rectangle is 2 less than three times the width. Find the dimensions of the rectangle if the area is 65 square meters.

11. The length of a rectangle is 7 meters less than twice the width. Find the dimensions if the area is 60 square meters.

12. The product of two consecutive integers is 56. Find the integers.

13. The product of two consecutive odd integers is 99. Find the integers.

14. Find two consecutive even integers such that the square of the smaller is 10 more than the larger.

15. The product of two consecutive odd integers is 1 less than twice their sum. Find the integers.

16. The product of two consecutive integers is three less than three times their sum. Find the integers.

17. The product of two consecutive even integers is 6 more than three times their sum. Find the integers.

18. The product of two consecutive odd integers is 77 more than twice the larger. Find the integers.

19. The product of two consecutive integers is 5 more than three times the larger. Find the integers.

20. Find three consecutive integers such that four times the sum of all three is 2 times the product of the larger two.

21. Find three consecutive integers such that three times the sum of all three equals the product of the larger two.

22. The medium side of a right triangle is 7 more than the shortest side. The longest side is 7 less than 3 times the shortest side. Find the length of the shortest side of the triangle.

23. One leg of a right triangle is one inch shorter than the other leg. If the hypotenuse is 5 inches, find the length of the shorter leg.

QUADRATIC EQUATIONS - WORD PROBLEMS