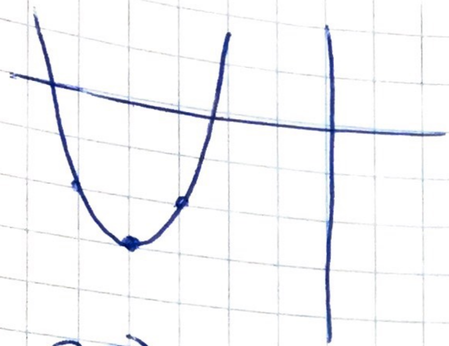
CHAPTER 4

24) Graph $y=(x+4)^2-3$ vertex form

LEFT 4 down 3



25) Graph $y=x^2-4x+4$ standard form

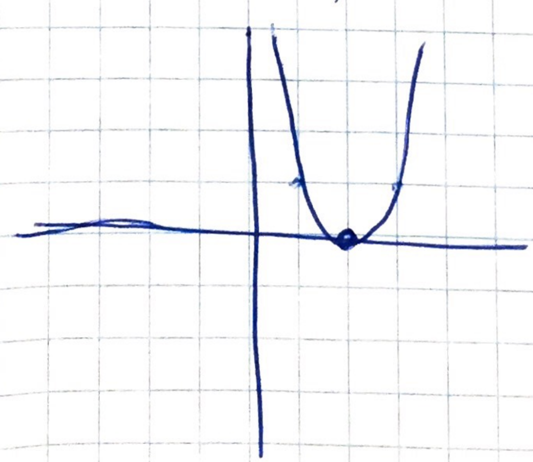
vertex $\frac{-b}{2a}=\frac{-(-4)}{2(1)}=\frac{4}{2}=2=h$

$k=2^2-4\cdot 2+4=4-8+4=0$

vertex $(2,0)$

$y=x^2-4x+4=(x-2)^2$ (Perfect square)

thus, this can be another way to come up with the vertex



26) Solve by factoring

$3x^2 + 27x + 54 = 0$

$3(x^2 + 9x + 18) = 0$ $\qquad 3 \cdot 6 = 18$

$3(x+3)(x+6) = 0$ $\qquad 3 + 6 = 9$

$\boxed{x = -3 \quad x = -6}$

27) Solve by factoring

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$\boxed{x = 3 \quad x = -3}$

28) Solve by completing the square

$x^2 + 10x + 15 = 0$ $\qquad a^2 + 2ab + b^2$

$+10 \quad +10$

$x^2 + 10x + \underline{25}$

$x^2 + 10x + 25 = 10$ $\qquad a = x \quad b = 5$

$\sqrt{(x+5)^2} = \sqrt{10}$

b^2 needs to be 25, right now it is 15

$x + 5 = \sqrt{10}$ or $x + 5 = -\sqrt{10}$

$\boxed{x = -5 + \sqrt{10} \quad \text{or} \quad x = -5 - \sqrt{10}}$

29) $x^2 - 7x = 9$

$x^2 - 7x - 9 = 0 \quad a=1 \;\; b=-7 \;\; c=-9$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{7 \pm \sqrt{(-7)^2 - 4\cdot 1(-9)}}{2\cdot 1}$$

$$= \frac{7 \pm \sqrt{49+36}}{2} = \frac{7 \pm \sqrt{85}}{2} = \boxed{\frac{7}{2} \pm \frac{\sqrt{85}}{2}}$$

30)

a) $\sqrt{-81} = \sqrt{81(-1)} = \sqrt{81} \cdot \sqrt{-1} = \boxed{\pm 9i}$

b) $\sqrt{-27} = \sqrt{27(-1)} = \sqrt{27} \cdot \sqrt{-1} = \sqrt{9 \cdot 3} \cdot \sqrt{-1}$

$= \boxed{\pm 3\sqrt{3}\, i}$

31) a) $i^5 = \underbrace{i \cdot i}\cdot\underbrace{i \cdot i}\cdot i$

$(-1)\cdot(-1)\cdot i = 1i = \boxed{i}$

b) $(2+2i) - (6-6i) = 2+2i-6+6i$

$= \boxed{-4+8i}$

c) $(-i)(8i) = -8i^2 = -8(-1) = \boxed{8}$

d) $(-2+4i)(-2-5i)$

$= 4 - 8i + 10i + 20$

$= \boxed{24+2i}$

	-2	$4i$
-2	4	$-8i$
$-5i$	$10i$	$-20i^2 = +20$

CHAPTER 5

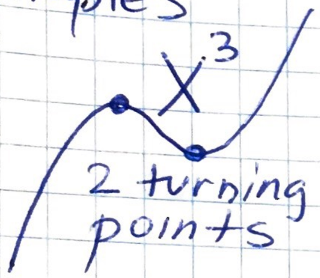
32) $8x^4 + 7x^3 + 5x^2 + 8$

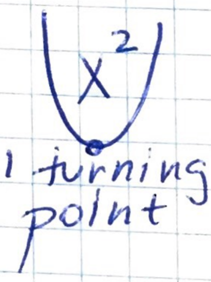
degree 4 means
4 TERMS means

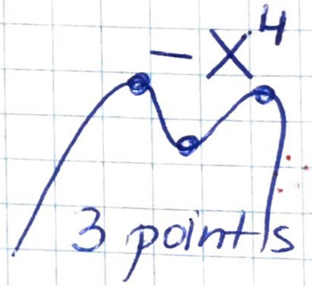
$\boxed{\text{QUARTIC polynomial of 4 TERMS}}$

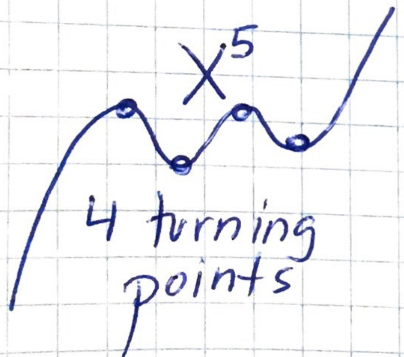
33) 4 turning points means degree is 5 (number of a degree is one more than the number of turning points)

EXAMPLES



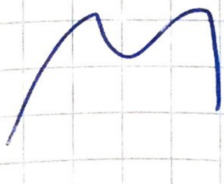






34) $y = -3x^4 - 9x^3 + x^2 + 1$

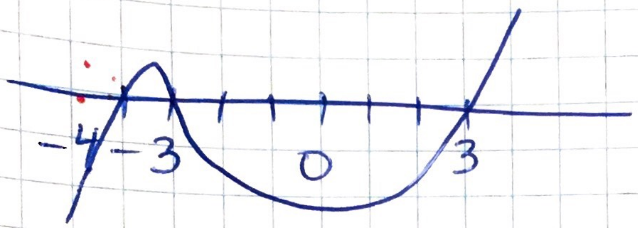
negative quartic (even degree means same direction)



$\boxed{\text{down and down}}$

35) $P(x) = (x+3)(x-3)(x+4)$

zeros: $-3, 3, -4$



3 factors means cubic

36) $P(5)=0 \quad P(2)=0 \quad P(-5)=0$

$P(x) = \underbrace{(x-5)(x-2)}(x+5)$

	x	-5
x	x^2	$-5x$
-2	$-2x$	10

$(x^2-7x+10)(x+5)$

	x	$+5$
x^2	x^3	$5x^2$
$-7x$	$-7x^2$	$-35x$
10	$10x$	50

$= \boxed{x^3-2x^2-25x+50}$

37) $64x^3-1=0$ difference of cubes

1 Real, 2 imaginary solutions

$(4x-1)(16x^2+4x+1)=0$

$\boxed{x=\frac{1}{4}}$

$$\frac{-4 \pm \sqrt{16-4\cdot16\cdot1}}{2\cdot16} = \frac{-4\pm\sqrt{16-64}}{32}$$

$$= \frac{-4\pm\sqrt{-48}}{32} = \frac{-4\pm4\sqrt{3}i}{32} = \boxed{-\frac{1}{8}\pm\frac{\sqrt{3}i}{8}}$$

38) $x^4 - 16 = 0$

$(x^2-4)(x^2+4)=0$

$(x-2)(x+2)(x^2+4)=0$

$\boxed{x=2 \quad x=-2}$ $\quad x^2=-4$

Real

$x = \pm\sqrt{-4}$

$\boxed{x=\pm 2i}$

imaginary

39) $4x^3+2x^2+3x+4$ divide by $x+4$

$$\begin{array}{r|l} & 4x^2 - 14x + 59 \\ x+4 & \cancel{4x^3} + 2x^2 + 3x + 4 \\ & -\cancel{4x^3} - 16x^2 \\ \hline & -14\cancel{x^2} + 3x + 4 \\ & +\cancel{14x^2} + 56x \\ \hline & 59x + 4 \\ & -59x - 236 \\ \hline & \boxed{-232} \end{array}$$

this means that $\dfrac{4x^3+2x^2+3x+4}{x+4} =$

$$4x^2-14x+59 - \frac{232}{x+4}$$

40) x^3+x^2-x+2 by $x+4$ using synthetic division

$$\begin{array}{r|rrrr} -4 & 1 & 1 & -1 & 2 \\ & \downarrow & -4 & 12 & -44 \\ \hline & 1 & -3 & 11 & \boxed{-42} \end{array}$$

$x^2-3x+11$ R

this means that: $\dfrac{x^3+x^2-x+2}{x+4} =$

$x^2-3x+11+\dfrac{-42}{x+4}$

41) Is $x-2$ a factor of $P(x)=x^3+2x^2-6x-4$?
If yes, what are the other two roots?

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & -4 \\ & \downarrow & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & \boxed{0} \end{array}$$

REMAINDER is 0, therefore $x-2$ is a factor (2 is a zero of $P(x)$, or $P(2)=0$)

x^2+4x+2

$a=1$
$b=4$
$c=2$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\frac{-4\pm\sqrt{16-4\cdot1\cdot2}}{2\cdot1}=\frac{-4\pm\sqrt{16-8}}{2}=\frac{-4\pm\sqrt{8}}{2}$$

$$=\frac{-4\pm2\sqrt{2}}{2}=\boxed{-2\pm\sqrt{2}}$$

the other two roots are irrational

42) RATIONAL root theorem
LIST All possible rational roots

$$x^3-6x^2+4x+9$$

FACTORS

$p=9;\ 1, 3, 9$

$q=1;\ 1$

Possible rational roots: $\pm1, \pm3, \pm9$

43) given roots: $3+\sqrt{6}$ and $2-\sqrt{5}$

Since these numbers are irrational, this means that there are also conjugate roots (conjugate root theorem)

$$3+\sqrt{6} \xrightarrow{\text{CONJ}} 3-\sqrt{6} \qquad 2-\sqrt{5} \xrightarrow{\text{CONJ}} 2+\sqrt{5}$$

$$(x-3-\sqrt{6})(x-3+\sqrt{6})(x-2+\sqrt{5})(x-2-\sqrt{5})$$

$$((x-3)^2-(\sqrt{6})^2)\cdot((x-2)^2-(\sqrt{5})^2)$$

$$(x^2-6x+9-6)(x^2-4x+4-5)$$

$$(x^2-6x+3)(x^2-4x-1)$$

	x^2	$-6x$	$+3$
x^2	x^4	$-6x^3$	$3x^2$
$-4x$	$-4x^3$	$24x^2$	$-12x$
-1	$-x^2$	$6x$	-3

$$x^4-10x^3+26x^2-6x-3$$

44) given roots $3+\sqrt{3}$ and $\frac{2}{5}$

irrational rational

conjugate theorem tells us there is one more irrational root $3\ \sqrt{3}$

therefore factors are

$$(x-3-\sqrt{3})(x-3+\sqrt{3})\left(x-\frac{2}{5}\right)$$

or to avoid fractions ↓ multiply by 5

$$(x-3-\sqrt{3})(x-3+\sqrt{3})(5x-2)$$

$$((x-3)^2-(\sqrt{3})^2)(5x-2)$$

$$(x^2-6x+9-3)(5x-2)$$

$$(x^2-6x+6)(5x-2)$$

	$5x$	-2
x^2	$5x^3$	$-2x^2$
$-6x$	$-30x$	$+12x$
$+6$	$30x$	12

$$\boxed{5x^3-32x^2+42x+12}$$

45) $-3x^3+x^2-x-6$

Positive roots

$P(x)=-3x^3+x^2-x-6$

2 sign changes means 2 or 0 (none) positive roots

NEGATIVE ROOTS

$P(-x)=-3(-x)^3+(-x)^2-(-x)-6$

$P(-x)=3x^3+x^2+x-6$

1 sign change means 1 negative root

CHAPTER 6

46) $\sqrt{200a^6b^7}=\sqrt{100\cdot 2a^6b^6\cdot b}$

$=10\sqrt{2}a^3b^3\sqrt{b}$

$=\boxed{10a^3b^3\sqrt{2b}}$

47) $\sqrt{72}+\sqrt{32}+\sqrt{18}=\sqrt{36\cdot 2}+\sqrt{16\cdot 2}+\sqrt{9\cdot 2}$

$=6\sqrt{2}+4\sqrt{2}+3\sqrt{2}$

$=\boxed{13\sqrt{2}}$

48) $\sqrt{2x-1}^2=3^2$ square both sides

$2x-\cancel{1}=9$ add 1

$\cancel{+1}\quad +1$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{10}{2}$ divide by 2

$\boxed{x=5}$

49) $3(x-2)^{\frac{3}{4}} = 24$ divide by 3

$(x-2)^{\frac{3}{4}\cdot\frac{4}{3}} = 8^{\frac{4}{3}}$ ← power of 4 ↖ cube root

$x-2 = 2^4$

$x-2 = 16$

$\boxed{x = 18}$

50) $(x+3)^{\frac{1}{2}} \cancel{-1} = x$ isolate parenthesis
$\cancel{+1} \quad +1$

$(x+3)^{\frac{1}{2}\cdot 2} = (x+1)^2$ square both sides

$\cancel{x}+\cancel{3} = x^2+2x+1$ subtract x and 3 to set equation equal to zero
$\cancel{-x}\ \cancel{-3} \qquad -x \quad -3$

$0 = x^2 + x - 2$

at this point you can try to factor or use MENU 5 and find roots

$0 = (x+2)(x-1)$

$\boxed{\cancel{x=-2} \quad x=1}$

x = -2 is an extraneous solution

check for extraneous solutions

$(-2+3)^{1/2} - 1 = -2 \rightarrow 1^{1/2} - 1 = -2$ FALSE ! $1-1=0$

$(1+3)^{1/2} - 1 = 1 \rightarrow 4^{1/2} - 1 = 1 \rightarrow 2 - 1 = 1$ ✓