HOMEWORK 5-5 and 5-8

5-5 (PART 1) RATIONAL ROOT THEOREM

1) a) $\overset{q}{x^2} + x - \overset{p}{2} = 0$

$p = 2$; FACTORS OF $2 = 2, 1$
$q = 1$; FACTORS OF $1 = 1$

possible rational roots: $\pm 1, \pm 2$
$(\pm p/q)$

TEST

$$\begin{array}{r|rrr} 1 & 1 & 1 & -2 \\ & \downarrow & 1 & 2 \\ \hline & 1 & 2 & \boxed{0} \end{array} \qquad \begin{array}{r|rrr} -1 & 1 & 1 & -2 \\ & \downarrow & -1 & 0 \\ \hline & 1 & 0 & -2 \end{array}$$

$$\begin{array}{r|rrr} 2 & 1 & 1 & -2 \\ & \downarrow & 2 & 6 \\ \hline & 1 & 3 & 4 \end{array} \qquad \begin{array}{r|rrr} -2 & 1 & 1 & -2 \\ & \downarrow & -2 & 2 \\ \hline & 1 & -1 & \boxed{0} \end{array}$$

the ROOTS are 1 and -2.

b) $4x^3 + 12x^2 + x + 3 = 0$

$p = 3$ FACTORS of $3 = 1, 3$
$q = 4$ FACTORS of $4 = 1, 2, 4$

possible rational roots:
$\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$
$\pm \frac{1}{4}, \pm \frac{3}{4}$

USE CALCULATOR (MENU 5 → ROOTS to "cheat" and find the actual ROOTS, then use synthetic division)

$$\begin{array}{r|rrrr} -3 & 4 & 12 & 1 & 3 \\ & \downarrow & -12 & 0 & -3 \\ \hline & 4 & 0 & 1 & 0 \end{array}$$

-3 is a rational root
$+\frac{\sqrt{2}}{2}$ and $-\frac{\sqrt{2}}{2}$ are imaginary roots

$4x^2 + 1 = 0$

$4x^2 = -1$

$\sqrt{x^2} = \sqrt{-\frac{1}{4}}$

$x = \pm\sqrt{-\frac{1}{4}}$

$x = \pm \frac{1}{\sqrt{2}} i = \boxed{\pm \frac{\sqrt{2}}{2} i}$

c) $3x^4 + 2x^2 - 12 = 0$

Fundamental theorem of Algebra says that there should be 4 roots (RATIONAL, irrational or imaginary)

$p = 12$ FACTORS of 12 = 1, 2, 3, 4, 6, 12

$q = 3$ FACTORS of 3 = 1, 3

(each p factor divided by each q factor gives the possible roots. All are + or −)

POSSIBLE RATIONAL ROOTS:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

USE CAlculator to determine actual rational roots, then show synthetic division, in order to find the remaining roots.

the roots in the calculator are -1.301635193 and 1.301635193 which are both "non-repeating" decimals (or irrational numbers). Therefore, none of the Listed possible rational roots are actually the roots. This means try to factor, or Quadratic formula

$3x^4 + 2x^2 - 12 = 0$ say that $x^2 = s$

$3s^2 + 2s - 12 = 0$ NOW it "looks like" quadratic

$s^2 + 2s - 36 = 0$ cannot be factored!

USE Quadratic formula

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(3)(-12)}}{2 \cdot 3}$$

$$= \frac{-2 \pm \sqrt{4 + 144}}{6} = \frac{-2 \pm \sqrt{148}}{6} = \frac{-\cancel{2} \pm \cancel{2}\sqrt{37}}{\cancel{6}} = \frac{-1 \pm \sqrt{37}}{3}$$

$s = x^2 = \frac{-1 + \sqrt{37}}{3}$; $x = \sqrt{\frac{-1 + \sqrt{37}}{3}}$ real or $s = x^2 = \frac{-1 - \sqrt{37}}{3} = \sqrt{\frac{-1 - \sqrt{37}}{3}}$ imaginary

5-5 (PART 2)

1)

a) $-10i$ by conjugate theorem we also have $+10i$ as the root

$(x+10i)(x-10i) = x^2 - (10i)^2$

difference of squares

$= x^2 - 100(-1)$

$= \boxed{x^2 + 100}$

b) 2 and $3i$, by conjugate theorem we also have $-3i$

$(x-2)(x-3i)(x+3i)$

$(x-2)(x^2 - (3i)^2) = (x-2)(x^2 - 9(-1))$

$= (x-2)(x^2+9)$

	x	-2
x^2	x^3	$-2x^2$
9	$9x$	-18

$= \boxed{x^3 - 2x^2 + 9x - 18}$

c) $-2i$, $\underline{+2i}$, $\sqrt{10}$, $\underline{-\sqrt{10}}$

$(x+2i)(x-2i)(x-\sqrt{10})(x+\sqrt{10})$

$(x^2 - (2i)^2)(x^2 - (\sqrt{10})^2)$

$(x^2 - 4i^2)(x^2 - 10)$

$(x^2+4)(x^2-10) = \boxed{x^3 + 4x^2 - 10x - 40}$

	x	$+4$
x^2	x^3	$4x^2$
-10	$-10x$	-40

5 - 5 (PART 2 continued)

2) a) $P(x) = x^2 + 5x + 6$

\# of positive real roots:

$P(x) = x^2 + 5x + 6$

NO sign changes

NO positive Real roots!

\# of negative roots:

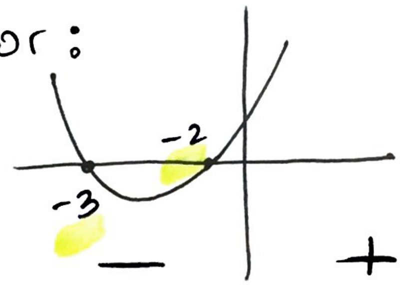
$P(-x) = (-x)^2 + 5(-x) + 6$

$P(-x) = \underbrace{+x^2 - 5x}\underbrace{\ + 6}$

2 sign changes

2 or 0 negative real roots!

check on calculator:
and circle the actual number of real roots.



b) $P(x) = 8x^3 + 2x^2 - 14x + 5$

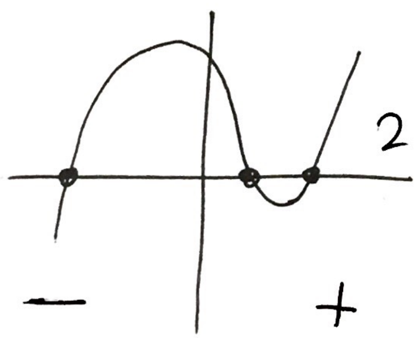
2 or 0 positive real roots

$P(-x) = 8(-x)^3 + 2(-x)^2 - 14(-x) + 5$

$P(-x) = -8x^3 + 2x^2 + 14x + 5$

1 negative real root!

calculator:



2 positive

1 neg

5-8 HOMEWORK

1) a) 3 points $(-1,8), (5,-4)$ and $(7,8)$

$3-1=2 \rightarrow$ quadratic model

$a=1$
$b=-6$
$c=1$

$y = 1x^2 - 6x + 1$ $\quad$ $\underline{\underline{r^2=1}}$ perfect fit

b) $(-1,-15), (1,-7), (6,-22)$

$3-1=2 \rightarrow$ quadratic model

$\underline{\underline{r^2=1}}$ perfect fit

$y = -1x^2 + 4x - 10$

c) $(-1,9), (0,6), (1,5), (2,18)$

4 points $\longrightarrow 4-1=3$ cubic model

$y = 2x^3 + x^2 - 4x + 6$ $\quad$ $\underline{\underline{r^2=1}}$ perfect fit

2)

year	price
1991 → 0	149
1995 → 4	158
2000 → 9	207

3 pts can be degree 2 or 1 (not higher!)

$\underline{\underline{L1}}$	$\underline{\underline{L2}}$
0	149
4	158
9	207

quadratic (x^2)

$y = 0.83\bar{8}x^2 - 1.10\bar{5}x + 149$

$\underline{\underline{r^2=1}}$ perfect fit

Linear (x)

$y = 6.58x + 142.81$

$r^2 = 0.904$ (not perfect)

Quadratic model is better!