HOMEWORK 7-5 / 7-6

1) $8^{2x} = 32$ since both 8 and 32 are powers of 2, we do not have to use logs

$8 = 2^3$ $\quad 32 = 2^5$

multiply

$(2^3)^{2x} = 2^5$

$2^{6x} = 2^5$

$6x = 5$

$\boxed{x = \frac{5}{6}}$

2) $7^n = 343$ $\quad 343 = 7^3$

$7^n = 7^3$

$\boxed{n = 3}$

3) $9^{2x} = 27$ $\quad 9 = 3^2$ $\quad 27 = 3^3$

$(3^2)^{2x} = 3^3$

$4x = 3$

$\boxed{x = \frac{3}{4}}$

4) $25^{2n+1} = 625$

$25^{2n+1} = 25^2$

$2n + 1 = 2$

$2n = 1$

$\boxed{n = \frac{1}{2}}$

5) $36^{-2x+1} = 216$

$(6^2)^{-2x+1} = 6^3$

$6^{2(-2x+1)} = 6^3$

$2(-2x+1) = 3$

$-4x + 2 = 3$

$-2 \quad -2$

$-4x = 1$

$\boxed{x = -\frac{1}{4}}$

6) $64^x = 4096$

$(4^3)^x = 4^6$

$3x = 6$

$x = 2$

7) $5^{2x} = 20$

$\log_5 5^{2x} = \log_5 20$

$2x = \log_5 20$

$\frac{2x}{2} = \frac{\log 20}{\log 5}$

$= 0.93$

method 2

$\log 5^{2x} = \log 20$

$2x \log 5 = \log 20$

$\frac{2x}{2} = \frac{\log 20}{\log 5}$

$x = 0.93$

8) $8^{n+1} = 3$

Method 1 (use $\log_8$)

$\log_8 8^{n+1} = \log_8 3$

$n+1 = \log_8 3$

$\boxed{n = \frac{\log 3}{\log 8} - 1}$

change of base: $\log_8 3 = \frac{\log 3}{\log 8}$

Method 2 (use log)

$\log 8^{n+1} = \log 3$

$(n+1)\frac{\log 8}{\log 8} = \frac{\log 3}{\log 8}$

$n+1 = \frac{\log 3}{\log 8} - 1$

$\boxed{n = \frac{\log 3}{\log 8} - 1}$

9) $\log_4 4^{n-2} = \log_4 3$

$n-2 = \log_4 3 + 2$

$\boxed{n = \frac{\log 3}{\log 4} + 2}$

10) $\log_4 4^{3n} = \log_4 5$

$3n = \frac{\log_4 5}{3}$

$n = \frac{\log_4 5}{3}$

11) $15^{2n-3} = 245$

$\log_{15} 15^{2n-3} = \log_{15} 245$

$2n-3 = \log_{15} 245 + 3$

$2n = \boxed{\frac{\log_{15} 245 + 3}{2}}$

12) $4^x - 5 = 12$

$+5 \quad +5$

$\log_4 4^x = \log_4 17$

$x = \boxed{\log_4 17}$

13) $\frac{2\log X}{2} = \frac{2}{2}$

$\log X = 1$

$10^1 = X$

$\boxed{X = 10}$

14) $\log(2X+5) = 3$

$10^{5} = 2X + 5$

$-5 \quad\quad -5$

$10{,}000 = 2X$

$\frac{9{,}995}{2} = \frac{2X}{2}$

15) $\log(3X-2) = 3$ ↑ error!

$10^{3} = 3X - 2$

$+2 \quad\quad +2$

$\frac{1002}{3} = \frac{3X}{3}$

$\boxed{X = \frac{1002}{3}}$

16) $\log(X-25) = 2$

$10^2 = X - 25$

$100 = X - 25$

$+25 \quad\quad +25$

$\boxed{X = 125}$

17) $\frac{2\log(2X+5)}{2} = \frac{4}{2}$

$\log(2X+5) = 2$

$10^2 = 2X + 5$

$100 = 2X + 5$

$-5 \quad\quad -5$

$\frac{95}{2} = \frac{2X}{2}$

$\boxed{X = \frac{95}{2}}$

18) $\frac{3\log(1-2X)}{3} = \frac{6}{3}$

$\log(1-2X) = 2$

$10^2 = 1 - 2X$

$-1 \quad\quad -1$

$100 = -2X$

-1

$\frac{99}{-2} = \frac{-2X}{-2}$

$\boxed{X = -49.5}$

19) $P = 8{,}100{,}000$

$r = 2\% = 0.02$

growth factor $1 + 0.02 = 1.02$

a) $A = P(1+r)^{t}$

$y = a \cdot b^{t}$

$\boxed{y = 8{,}100{,}000(1.02)^{t}}$

b) $A = 9{,}000{,}000 \quad t = ?$

$9{,}000{,}000 = 8{,}100{,}000(1.02)^{t}$

$\frac{90}{81} = \frac{81(1.02)^{t}}{81}$

$\log \frac{10}{9} = \log 1.02^{t}$

$\frac{\log \frac{10}{9}}{\log 1.02} = \frac{t \log 1.02}{\log 1.02}$

$5.32 = t$ It would take about 5.32 years to reach 9 million

20) $\frac{3}{3}e^{3x-5} = \frac{49}{3}$

$\ln e^{3x-5} = \ln \frac{49}{3}$

$3x - 5 = \ln \frac{49}{3} + 5$

$+5$

$\frac{3x}{3} = \boxed{\frac{\ln \frac{49}{3} + 5}{3}}$

21) $\frac{7}{7}e^{5x+8} = \frac{0.23}{7}$

$\ln e^{5x+8} = \ln \frac{0.23}{7}$

$5x + 8 = \ln \frac{0.23}{7} - 8$

-8

$\frac{5x}{5} = \boxed{\frac{\ln \frac{0.23}{7} - 8}{5}}$

22) $6 - e^{12x} = 5.2$

$-6 \quad -6$

$-e^{12x} = -0.8$

$\ln e^{12x} = \ln 0.8$

$\frac{12x}{12} = \boxed{\frac{\ln 0.8}{12}}$

23) $-7 + \ln 2x = 4$

$+7 \quad +7$

$\ln 2x = 11$

$e^{11} = 2x$

$x = \boxed{\frac{e^{11}}{2}}$

24) $3 - 4\ln(8x+1) = 12$

$-3 \quad -3$

$\frac{-4\ln(8x+1)}{-4} = \frac{9}{-4}$

$\ln(8x+1) = -\frac{9}{4}$

$e^{-9/4} = 8x + 1$

$-1 \quad -1$

$\frac{e^{-9/4} - 1}{8} = \frac{8x}{8}$

$x = \boxed{\frac{e^{-9/4} - 1}{8}}$

25) $\ln x + \ln 3x = 14$

$\ln 3x^2 = 14$ Product prop.

$\frac{e^{14}}{3} = \frac{3x^2}{3}$

$\frac{e^{14}}{3} = x^2$

$x = \boxed{\sqrt{\frac{e^{14}}{3}}}$

26) $2\ln x + \ln x^2 = 3$

$\ln x^2 + \ln x^2 = 3$

$\ln x^4 = 3$

$e^3 = x^4$

$\boxed{x = \sqrt[4]{e^3}}$

27) $\ln x + \ln 4 = 2$

$\ln 4x = 2$

$e^2 = 4x$

$\frac{e^2}{4} = \frac{4x}{4}$

$\boxed{x = \frac{e^2}{4}}$

28) $\ln x - \ln 5 = -1$

$\ln \frac{x}{5} = -1$

$5 \cdot e^{-1} = \frac{x}{5} \cdot 5$

$\boxed{x = 5e^{-1}}$

29) $\ln e^{x+5} = 17$

$x + 5 = 17$

$-5 \quad -5$

$\boxed{x = 12}$

30) $y = a e^{-0.00012t}$

$32 = 100 e^{-0.00012t}$

↑ remaining % ↑ beginning %

$\frac{32}{100} = e^{-0.00012t}$

$\ln 0.32 = \ln e^{-0.00012t}$

$\frac{\ln 0.32}{-0.00012} = \frac{-0.00012t}{-0.00012}$

$t = 9495.29$

The approximate age of the bone is 9495 years.